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Ancient
VEDIC MATHEMATICS

PUSHP-3

Mathematics with Smile

-: वैदिक गणित सूत्र :-

१ एकाधिकेन पूर्वेण
२ निखिलं नवतश्चरमं दशतः
३ ऊर्ध्वतिर्यग्भ्याम्
४ परावर्त्य योजयेत्
५ शून्यं साम्यसमुच्चये
६ (आनुरुप्ये) शून्यमन्यत्
७ संकलनव्यवकलनाभ्याम्
८ पूरणपूरणाभ्याम्
९ चलनकलनाभ्याम्
१० यावदूनम्
११ व्यष्टिसष्टिः
१२ शेषाण्यङ्केन चरमेण
१३ सोपान्त्यद्वयमन्त्यम्
१४ एकन्यूनेन पूर्वेण
१५ गुणितसमुच्चयः
१६ गुणकसमुच्चयः

By :
DR. NARINDER PURI

A:3:2

R
510
PUR-A.3:2

वेदात सर्वं प्रसिद्धयति ।

वेद सम्पूर्ण ज्ञान के अनादि अनन्त स्रोत ।

Vedas are the Eternal Source of Knowledge !

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Foundation

Gurukul Ka

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गुरुकुल कांगड़ी विश्वविद्यालय
कृपया पुस्तक के ऊपर कोई निशान आदि
न लगायें।

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ANCIENT

VEDIC MATHEMATICS

MATHS WITH SMILES

Based on

SIXTEEN SIMPLE SUTRAS

FROM THE VEDAS

PUSHP - 3

DONATION

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प्रातः प्रभृति सायान्तम्
सायन्तः प्रातरन्तम्
यत् करोमि जगन्नाथ
तदस्तु तव पूजनम्

From morning till eve,
And evening till morn,
Whatever I do ,
Oh, Lord of Universe
Let that be my prayar of thee !

* * * * * * * * * * * * * * * * *

A
Humble Offering
at the
Lotus feet
of
Jagad Guru Shankra Charya
Swami **Bharti Krishen Tirthji Maharaj**
who has revived VM

* * * * * * * * * * * * * * * * * * * *

Vedic Mathematics provides very easy, one line, mental and superfast methods alongwith magic speed cross checking system.As such Vedic Maths is a boon for all competitions. Vedic methods are easy to understand and their variety, speed and ease brings joy in the heart and smile on the face of the students. The Vedic Maths always provides a general method which is always faster than the present system but Vedic system also provides various other methods which often work with magic speed and very frequently the students of Vedic maths class move faster than the teacher and speak out the answer as soon as the problem is written on the black board.

Prof. K.R. Williams of London says, "This element of choice in the Vedic System, even of innovation, together with mental approach, brings a new dimension to the study and practice of Mathematics. The variety and simplicity of methods brings fun and amusement, the mental practice leads to a more agile, alert and intelligent mind and innovation naturally follows".

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THE SIXTEEN SUTRAS OF VEDIC MATHEMATICS

सूत्र :-

1 एकाधिकेन पूर्वेण	1. Ekadhikena Purvena (also a corollary)
2 निखिलं नवतश्चरमं दशतः	2. Nikhilam Navatascaramam Dasatah
3 ऊर्ध्वतिर्यग्भ्याम्	3. Urdhva-tiryagbhyam
4 परावर्त्य योजयेत्	4. Paravartya Yojayet
5 शून्यं साम्यसमुच्चये	5. Sunyam Samyasamuccaye
6 {आनुरूप्ये} शून्यमन्यत्	6. (Anurupye) Sunyamanyat
7 संकलनव्यवकलनाभ्याम्	7. Sankalana-vyavakalanabhyam (also a corollary)
8 पूरणपूरणाभ्याम्	8. Puranapuranabhyam
9 चलनकलनाभ्याम्	9. Calana-Kalanabhyam
10 यावदूनम्	10. Yavadunam
11 व्यष्टिसमष्टिः	11. Vyastisamastih
12 शेषाण्यङ्केन चरमेण	12. Sesanyankena Caramena
13 सोपान्त्यद्वयमन्त्यम्	13. Sopantyadvayamantyam
14 एकन्यूनेन पूर्वेण	14. Ekanyunena Purvena
15 गुणितसमुच्चयः	15. Gunitasamuccayah
16 गुणकसमुच्चयः	16. Gunakasamuccayah

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-: VEDIC MATHEMATICS SUTRAS :-

1. By one more than the previous one
2. All from nine & last from ten
3. Vertically & Crosswise
4. Transpose & Apply
5. The summation is equal to zero
6. If one is in ratio, the other one is zero
7. By addition and subtraction
8. By the completion and noncompletion
9. Sequential motion
10. The deficiency
11. Whole as one & one as whole
12. Remainder by the last digit
13. Ultimate and twice the penultimate
14. By one less than the previous one
15. The whole product is the same
16. Collectivity of multipliers

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SUB SUTRAS OR COROLLARIES

1 आनुरूप्येण	Anurupyena
2 शिष्यते शेषसंज्ञः	Sisyate Sesasamjnah
3 आद्यमाद्येनान्त्यमन्त्येन	Adyamadyenantyamantyena
4 केवलैः सप्तकं गुण्यात्	Kovalalh Saptakam Gunyat
5 वेष्टनम्	Vestanam
6 यावदूनं तावदूनम्	Yavadunam Tavadunam
7 यावदूनं तावदूनीकृत्य वर्ग च योजयेत्	Yavadunam Tavadudunikrtya Varganca Yojayet
8 अन्त्ययोर्दशकेऽपि	Antyayordaskepi
9 अन्त्ययोरेव	Antyayoreva
10 समुच्चयगुणितः	Samuccayagunitah
11 लोपनस्थापनाभ्याम्	Lopanasthapanabhyam
12 विलोकनम्	Viloknam
13 गुणितसमुच्चयः - समुच्चयगुणितः	Gunitasamuccayah Samuccayagunitah
14 द्वन्द्वयोग	Dwandwa Yoga
15 शुद्धः	Shudha
16 ध्वजकं	Dhwajakam

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SWAMI BHARATI KRISHNA TIRTHA

Swami Bharati Krishna Tirtha, author of the first recent Vedic Mathematics book was the Shankaracharya of Goverdhan Peeth. Jagadguruji, named Venkatraman in his early days, was an exceptionally brilliant student and invariably won the first place in all the subjects in all the classes throughout his educational carrier.

After winning the highest place in the B.A. examiation, Shri Venkatraman Saraswati appeared at the M.A. examination of the American college from Bombay Centre in 1903. In 1904, at the age of just twenty, he passed M.A. examination in further seven subjects simultaneously securing the highest honours in all, which is perhaps the all time world-record of academic brilliance. His subjects included Sanskrit Philosophy, English, Mathematics, History and Science.

In the words of Swamiji,

"The Sutras (aphorisms) apply to and cover each and every chapter of each and every branch of mathematics (including arithmatic, algebra, geometry- plane and solid, trigonometry -plane and spherical, conics- geometrical and analytical, astronomy, calculus etc. etc.). In fact, there is no part of mathematics, pure or applied, which is beyond their jurisdiction. The Sutras are easy to understand, easy to apply and easy to remember; and the whole work can be truthfully summarised in one word "mental",".

Even as regards complex problems involving a good number of mathematical operations (consecutively or even simultaneously to be performed), the time taken by the Vedic method will be a third, a fourth, a tenth or even a much smaller fraction of the time required according to modern (i.e. current) Western methods.

And, in some very important and striking cases, sums requiring 30, 50, 100 or even more numerous and cumbrous "steps" of working (according to the current Western methods) can be answered in a single and simple step of work by the Vedic method! And little children (of only 10 or 12 years of age) merely look at the sums written on the blackboard (on the platform) and immediately shout out and dictate the answers from the body of the convocation hall (or other venue of the demonstration). And this is because, as a matter of fact, each digit automatically yields its predecessor and its successor! and the children have merely to go on tossing off (or reeling off) the digits one after another (forwards) by mere mental arithmetic (without needing pen or pencil, paper or slate etc)!

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Whatever we see in nature has a perfect mathematical structure.The rotation of the earth, movement of the stars and planets, growth of the plants and trees etc.are all perfectly mathematically structured.The blooming of a flower, the growth of a plant, the rotation of the earth, all go on so very effortlessly.Recent research is bringing to light the perfect mathematical structure of the Vedic texts. As a matter of fact whatever we do in life has mathematics in some form or the other. It is not possible to prepare even a delicious cup of coffee without a precise sense and application of mathematical proportion of different ingredients used. It's a pity that such a wonderful subject of fundamental and vital importance - mathematics -is creating the problem of 'MATHS ANXIETY' in all parts of the world. The common students are afraid of Mathematics. The gravity of the problem is quite clear from a recent survey report published in America. The top 10% of the students of Maths were asked the question, "Do you like Mathematics" ,And 60% of them said No. The answer of the remaining 90% is quite obvious.

Further, whatever we want to do, we always have a large number of alternatives and choices in doing the same.How strange that in Mathematics there seems to be almost no choice, and, most of the time we have to follow a unique, fixed, monotonus, often boring and tedious procedure in whatever arithmetical operation we want to do. Interestingly enough this unique approach is considered to be a quality by some mathematicians. Anything fixed, naturally becomes monotonus and boring and Maths Anxiety is the natural result. Vedic Maths on the other hand with its multiple choices makes maths a playful and interesting subject, which naturally brings frequentsmiles on the face and joy in the heart of the students.

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PREFACE

Vedic Mathematics system has been revived in our times by Jagadguru Shankracharya Swami Bharati Krishen Tiratha Ji.

The sixteen Vedic sutras provide a structured system which is applicable in, and, hence, is capable of explanining almost any facet of cosmos,both at abstract as well as applied level.These sutras in their philosophical interpretation even explain the functioning of human psychology. In their simple applied form, their crude translation in English has been presented in the beginning.

Sanskrit is a rich language and for deeper meaning one has to dive deep into the original sutras. The first Vedic Sutra' Ekadhiken Purvena' in simple English means by one more than the previous one, and a couple of very interesting and wonderful applications are presented in Pushp I. However the same sutra Ekadhiken Purven (एकाधिकेन पूर्वेण) explains the whole concept of growth and development of an individual, society or any facet of knowledge. The word purven essentially means the previous state. Ekadhiken means one higher than (the previous state). The higher state again becomes the previous and we go higher and higher and higher step by step, and that speaks out the whole process of growth and development. Similarly each sutra has a deep philosophical meaning. However, in the initial Pushps we are only looking at the simple applied facets of these sutras.

In a NUTSHELL, Vedic Maths (VM) offers a new and entirely different approach to mathematics based on pattern recognition for resolving the current crisis in Maths education. VM allows for constant expression of a student's creativity and has been found to be easier to learn. For any problem there is always one general technique applicable to all cases.There are also a number of special techniques for special pattern problems. The element of choice and flexibility at each stage keeps the mind lively and alert and develops clarity of mind and intuition, and thereby Holistic development of the human brain automatically takes place.

The real beauty of the VM is that as you start using this system, new links automatically come to the mind. A large number of persons, mostly students, from remote corners of mother India have helped me in reviving the new links. (The chapters 1,2,3,4,5,and 13 are beyond the basic book).

This whole project is being done with the spirit of 'Gyana Yajna'. Thanks are due to a large number of selfless workers for their labour of Love. I would like to specially thank Dr. P.R.Sharma, and Kumari Pragati Goel for their kind help in drafting. I am thankful to Shri Ramesh Chander for

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loading it on P.C.

Just like the superfast speed of working of Vedic Mathematics, it is spreading around the whole world with the unimaginable speed.Vedic Maths is being received with a great hope, joy and smile in all parts of the world.

It is a matter of great joy that the members of **Maharishi International Organisation** are translating the **Vedic Mathematics PUSHPMALA in various foreign languages.**

I pray to the Divine that this gift of the Veda may help you to develop intuition capability, and inspire you to carry the fragrance of this Maths of Smiles to all those around you.

N.PURI

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"ANCIENT VEDIC MATHEMATICS"

SALIENT FEATURES

A Complete System

Ideal for Holistic Development of Human Brain

- 16 Vedic sutras of 120 words can solve all mathematical problems.
- With a firm scientific footing
- A revolution in concept of learning
- Opening new areas of R & D
- The Vedic Maths can be used to check the computer results, and frequently supersedes calculators.
- Revealing hither-to-unknown,Great and Mysterious Qualities of Numbers.
- Simple, short mental mathematical processes which often work with magic speed.
- Requires no prerequisite courses
- Mathematics otherwise burden, turns to be a joyful and playful subject.
- With multiple choice procedures
- A boon for all competitions like Banking, Engineering, Medical, Economics, UPSC, ONGC, MBA, etc., where the use of calculator is not allowed.
- More useful, accurate and applicable at times, compared to modern computers.
- The speed of learning different facets of mathematics is at least four to five times faster than that required in the present system.
- Perfectly in tuned with the Vedic tradition of minimizing the efforts and maximizing the results.
- Students of VM frequently experience the bliss within.

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VEDIC MATHEMATICS NEWS

"It is tempting to think of them (Indians) as computer scientists ....."Let us not forget that among the great accomplishments of the Indian thinkers were the invention of zero, and of the Binary number system a thousand years before the West". Dr. R. Briggs of NASA, California, in Jl. of Artificial Intelligence, Spring, 85.

It is interesting to note that this Ancient Indian Binary number system is used in all the calculators and computers of the whole world.

1. The first book of our times on Vedic Mathematics was written by H.H. Swami Bharati Krishen Tirth Ji, Shankaracharya of Puri (1884-1960) in English. This book is already running in 11th reprint in last 20 years. It has already been translated in Marathi in 4 volumes and Gujrati. Further, it is being translated in Hindi, Latin and German.

2. The Symposium and Workshop have been organised at Nagpur, London, Roorkee and B.H.U. Varanasi in 1985, 1986 and 1987 respectively. Further, workshops and seminars have been organised at Rohtak, Pantnagar, Jaipur,Ahmedabad, England, U.S.A., Mexico, Norway, Sweden, Denmark, Hungary, Italy, Spain, France, Germany, Switzerland and Holland in 1988.

3. The Vedic Mathematics is being taught at the School of Economic Sciences, St. James School, London;MIU, Maharishi's schols at Washington, Iowa, Holland etc.

4. Swami Ji's book introduces the wonderful applications to Arithmetical computations, theory of numbers, compound multiplications, algebraic operations, factorisations, simple quadratic and higher order equations, simultaneous quadratic equations,partial fractions, check, squaring, cubing, square root, cube root, coordinate geometry and the wonderful Vedic Alpha-Numerical code.

5. Recently a course was run at the Mary Ward Centre; London demonstrating that 'Vedic Methods' can be used for entire G.C.E. A-level mathematics. This is equivalent to our 10 + 2 level in India.

6. On this novel Indian system of Mathematics futher research work is being done at Roorkee University, America and U.K.. The research includes applications of Vedic Mathematics to Algebra,trigonometry, three dimensional coordinate geometry, solution of plain and spherical triangles, calculus, linear and nonlinear differential equations, matrices and determinants, and log and exponentials,computer graphics and software.

7. The most interesting point to note is that the Vedic

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Mathematics provides unique solution in several instances where only trial and error method is available at present. For example the Kepler's equation can be solved in just 90 seconds as compared to the present tedious trial and error method, which takes nearly half an hour.

LIST OF VEDIC MATHEMATICS BOOKS AND LITERATURE

1. 'Vedic Mathematics' English by Swami Bharati Krishen Tirth Ji.

2. Vedic Ganit in Gujrati, by Dr.N. Kansara.

3. 'Vedic Ganit' by Dileep Kulkarni in Marathi,4 parts

4. Ancient Vedic Mathematics -PUSHP-I and PUSHP-II by N. Puri (Hindi/English). The only books available in Hindi.

5. Glimpses - Vedic Maths, by Dr.N. Puri (Hindi/English)

6. Notes of Workshop on "Vedic Mathematics and its Computational Potentialities"based on lecture notes of Roorkee Workshop,Aug., 87. G.Editor N. Puri (English)

7. Workshop on Vedic Mathematics by Dr. N.Puri at B.H.U. Varanasi, Dec. 1987

8. 'Overview of Vedic Mathematics', by Dr. N. Puri; Workshop on Vedic Mathematics, at University of Rajasthan, Jaipur sponsored by Ministry of Education, Govt. of India; also presented at international universities in 18 countries.

9. Lecture Notes of Elementary Vedic Mathematics, by Dr. N. Puri,International courses held at Switzerland,Mexico and America in June and July, 1988.

10. 'Higher Applications of VM' English by Dr. N. Puri.

11. Vedic Mathematics three month correspondence course, Part I, Dr.N.Puri offered by SSG,Roorkee.

12. Vedic Mathematics three month correspondence course, Part II Dr. N. Puri, offered by SSG, Roorkee.

13. 'Introductory Lectures on Vedic Mathematics', by A.P. Nicholas,J.R. Pickles and K.Williams

14. 'Discover Vedic Mathematics', By K.Williams

15. 'Triples', by K.Williams

16. 'Vertically and Crosswise', by A.P. Nicholas, K.Williams and J.R. Pickles

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17. Maths or Magic by J House

18. Vedic Ganitam, by T.S.Raju

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WORK AT ROORKEE

A high level research committee has been formed at Roorkee University to plan and do further research and development of Vedic Mathematics. The committee has senior faculty members of Mathematics, Civil Engineering, Mechanical Engineering, Hydrology, Earthquake Engineering, Metallurgical Engineering, and Electronics and Computer Engineering, departments as its members. Major research proposals are under preparation. Already,the Spiritual Study Group consisting of the students and staff members of Roorkee University has been doing humble efforts towards the research, development and spreading of this wonderful mathematical facet of our rich, vast and scientific cultural heritage.

Video Cassettes: A series of Video cassettes of Lectures on different facets of Vedic Mathematics are being produced in different parts of the world. These are available on sale. The interested persons or institutions may send Rs. 5/- for full details of contents of each cassette, rules and regulations of purchase system etc.

Lectures: Thousands of Vedic Mathematics demonstration lectures are being delivered in different parts of the world. In India, hundreds of lectures have already been given in Delhi, Punjab, Gujrat,West Bengal, UP, MP, Karnataka, Kerala, A.P., Goa,Rajasthan, Haryana and Maharashtra.Further, demonstration lectures and seminars on Vedic mathematics have already been delivered by the author in several cities of America, England, Canada, Mexico, Norway, Sweden,Finland, Denmark, Hungry, Germany, Italy, Switzerland,Spain and Holland in 1988.

Courses: Starting with the first course in May, 1985, SSG has already organised VM courses for 44 batches attended by 4400 participants ranging from school children, teachers, university students, research scholars, field engineers to the professors of the universities of India, U.S.A., Canada, Mexico, Holland, France, England, Germany, Switzerland, Spain, Panam etc. by Dec.,1988. All of them enjoyed learning the simple quick and blissful methods of solving the otherwise tedious mathematical calculations.

Institute of Vedic Sciences-Correspondence Course

On repeated requests from persons residing outside Roorkee, the international correspondence course for home study of the wonderful and magic speed'Vedic Mathematics' has been started from the auspicious Maha Kumbh, Baisakhi (April 14,1986). With a view to revive the ancient sciences, SSG has started the Institute of Vedic Sciences at Roorkee. Presently the institute is organising correspondence courses of six months duration in Hindi and English. New batches start from auspicious Baisakhi (April) and Dushera (October)

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every year.

Anybody who is in class VIII and above can join these courses.Fortnightly lessons, containing a large number of solved problems, and examples are sent to every participant. The students are asked to submit solution of the problems, which are duly checked. A certificate is awarded at the end of the course, after a postal test.The detailed information, course content, glimpses of Vedic Maths. etc. are available for Rs. 10/- (Further add Rs. 7.00 if desired by registered post) for India and $ 3 for other countries.

WORKSHOP

A symposium on Vedic Mathematics was held in Feb. 1985 at Nagpur.The second at Bhartiya Vidya Bhawan's London centre in Sept. 1986. The first 11-day workshop on Vedic Maths and its computational potentialities was held at University of Roorkee in Aug. 1987.Further Vedic Maths workshops have been organised at BHU Varanasi,Pantnagar University, Rajasthan University, Jaipur, Delhi,Ahmedabad and Bangalore. Vedic Maths seminars have also been held (during April,1988) at Imperial College, London; Oxford University, Cambridge University, and Ideal Village, Lancashire; Washington; Sidewell Friends School, Washington; Harvard University, Scientific systems, Boston; Annual NCTM meeting at Chicago; Los Angles; Stanford University, Pola Alto; University of California, Berkeley; Indiana; Goldendome MI University, Fairfield; Toronto; Mexico University, MIU Oslo, Academy at Liley Hamer,Norway, Upsala University and Stockholm University, Sweden; Turku University, Finland; Copenhagen University and H.C. Oersted Institute ,Denmark; Hemburg University and Technical University, Berlin; Institute of Leadership Budapast ,Hungary. Further during the month of May,1988 Vedic Maths seminars were held at Main University Vienna, Austria; TM Centre Rome; Engineering faculty- University of Naples and University of Milan, Italy; MERU Seelisberg, Geneva University, Switzerland; Institute Henry Poincane Paris,France; Barcelona; Spain, MERU Vlodrop, Amsterdam University, Holland.The first series of International trainers training programme of Vedic Mathematics teachers was organised at MERU Seelisberg, Switzerland, Washington, Mexico and MIU Fairfield, America, during June-July 1988. All the lectures were delivered by the author. All these courses were video recorded. Systematic teaching of Vedic Mathematics hase already been started in several schools of America. The initial response of teaching VM is excellent.

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Ancient Vedic Mathematics:

With a view to provide the comprehensive details of this wonderful, Ancient Bhartiya Cultural Heritage, SSS is planning to publish a series of books called Pushpmala.

SSS most prayerfully and humbly has already published Pushp-1 and 2 in English & Hindi and Pushp-3 is being published in English. Further higher applications of VM has also been published which includes applications in trigonometry, coordinate geometry & calculus.

Ancient Vedic Mathematics New Links

As a result of the humble research efforts of the members of SSG, a few of the missing links of the Ancient Vedic Mathematics have already been revived.Specifically the superfast Vedic methods of addition, subtraction, compound addition and subtraction, tables,compound multiplication, extensions of the base system for multiplication, multiplication of three or more numbers in one line, squaring of very large numbers, the magic speed methods of checking the correctness of any arithmetic or algebraic addition, subtraction, multiplication, division, factorisation, etc.have already been re-established. Further using the Vedic Tribhujank approach, the working of problems in Trigonometry both plane and spherical as well as coordinate geometry (2D & 3D) is drastically simplified. Many of the new links alongwith multiplication, division, fractions etc.are being published in Pushp-1 and 2 and 3, and in the Higher Applications of VM .Further links shall be published in future Pushp mala.

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Opinion of the Participants:

Everybody who has learnt Vedic Mathematics enjoyed bliss during the process of learning and has been thrilled by the superfast, almost magic speed calculation procedures of the Vedic Mathematics. The Spiritual Study Group at Roorkee has been organising Elementary and Advance courses on Vedic Mathematics, since May,85. All the participants from different parts of the world are asked to fill up the response forms comparing the present system of Mathematics with the Vedic system. A total of 11 questions were asked.The minimum score in favour of Vedic Mathematics for any question in any course so far was 80%. Most of the questions got 100% favour. Further, comments of participants are motivating and inspiring for spreading Vedic Mathematics all over the world.

A few glimpses of the participants opinions:

- A boon for all competitive examinations
- Could never imagine it would be so interesting
- Very pleasant to have this course at the university
- How advanced our ancestors were even in Mathematics
- Want to learn advanced topics
- Increases devotion towards our INDIA and its ancient culture
- Now I wish to learn Sanskrit to know more about our ancient scientific developments
- Every person on earth should have the knowledge of Vedic Mathematics
- It was a boon to learn Vedic Mathematics in Dushera holidays (I had planned for my regular studies)
- Want to learn more about Vedic Mathematics like factorisation etc. which are difficult in modern system
- Amazing speed, very wide applications covering almost all the mathematical operations
- A dynamic step for its introduction in educational system is a must
- It can be of great use for children if made part of syllabus
- Mysterious nature of numbers.After little practice problems can be mentally solved at magic speed

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- Very educative, has much scope for coming generation to start its practice from beginning

- It is a strong proof of our rich cultural heritage

- By Vedic methods Maths , which is a burden, becomes a joy for the children

- This is the most interesting and thrilling time of my life. Being a mathematics teacher I have interest in maths, but this Vedic Mathematics has thrilled me. When I come in contact with the Vedic Mathematics it gave such a pleasure which I can't explain in words.

A teacher of University Adarsh Bal Niketan

- The lectures were eye openers and most thrilling

- A glimpse of the lofty trakking in Himalayas of ancient knowledge in Maths,developed with spiritualism.Cudos to SSG.

A professor of University

- I have enjoyed all aspects of Vedic Maths course - beautifully blissful- two weeks.

Mrs. R.Leahy Iowa U.S.A.

- A very enjoyable course - Looking forward to learning deeper applications.

S.Olessen, Sweden

- I enjoyed practising Vedic Maths as much as I enjoy Meditating.

R. Kneedler

- Teachers of American schools report

i) the whole experience of teaching VM created bubbles of bliss and joy in the class.

ii) There faces seem to light up when they do VM.

iii) You have several alternatives and you can choose the easiest last year the maths was a little boring but with VM it is easier.

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CHAPTER 1

ADDITION USING COMPLEMENTS OF NUMBERS

1.1 INTRODUCTION

We have already discussed in Pushp 1 the addition of numbers by using the subsutra 'Shudha'.This was quite simple but sometimes in this procedure we had to add larger digits which might not be convenient for atleast some of us, i.e., second approach of addition in Vedic system is even simpler than the already explained Shudhikaran process. This method uses the complements of numbers. In other words it uses the Nikhilam sutra (All from nine and last from ten) to reduce the magnitude of the digits larger than 5. Thus addition can be done using numbers only upto 5 and always playing within 0 to 9 range of digits.

Let us demonstrate the procedure with the help of some examples.

1.2 Example 1:

$$\begin{array}{rr} & 9 \\ + & 8 \\ \hline & \end{array}$$

Step 1: In the first step all the numbers which use larger than 5 will be converted to their complement by the use of Nikhilam sutra.

Thus $9 = 1\ \bar{1}$

and $8 = 1\ \bar{2}$

Step 2: The addition is carried out column by column in usual manner, moving from bottom to top or top to bottom.However, instead of requiring to add 9 to 8 now we have to add only $\bar{1}$ to $\bar{2}$. Thus $\bar{1} + \bar{2} = \bar{3}$. Write this $\bar{3}$ at the unit place.

$$\begin{array}{rrr} & 1 & \bar{1} \\ + & 1 & \bar{2} \\ \hline & & \bar{3} \end{array}$$

Step 3: Now add the digits of the next higher level 1 + 1 = 2.

$$\begin{array}{rrr} & 1 & \bar{1} \\ + & 1 & \bar{2} \\ \hline & 2 & \bar{3} \end{array}$$

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Step 4: Thus addition of 9 and 8 is 2 $\bar{3}$ which can be normalised again by employing Nikhilam sutra and 2 $\bar{3}$ = 17. Complement of $\bar{3}$ is 7 and reduce the previous digits by 1 (2 - 1 = 1).

Let us take the next example a little more complicated.

1.3 **Example 2:**

$$\begin{array}{r} 9\ 6\ 4 \\ +\ 8\ 7\ 6 \\ \hline \end{array}$$

Step 1: Convert all the big digits into their vinculum form.

$$1\ 0\ \bar{4}\ 4$$

$$1\ \bar{1}\ \bar{2}\ \bar{4}$$

We shall be careful in applying the Nikhilam sutra; all from nine and last from ten. Thus, the last digits in this problem are 6 and 6 respectively. In other words, in applying the Nikhilam sutra to find complements we always proceed from left to right, i.e. 9 - 9 = 0, 10 - 6 = 4 and so on.

Also note that it is not required to find the complement of the first digit 4 in the upper row, as it is smaller than 5.

Step 2 : Add column by column from right to left 4 + $\bar{4}$ = 0, will appear in the right most column.

$$\begin{array}{r} 1\ 0\ \bar{4}\ 4 \\ 1\ \bar{1}\ \bar{2}\ \bar{4} \\ \hline 2\ \bar{1}\ \bar{6}\ 0 \end{array}$$

$\bar{4} + \bar{2} = \bar{6}$ at the next higher level. $0 + \bar{1} = \bar{1}$ in the next higher level and 1 + 1 = 2 the left most column. It is interesting to note that most of the times we do not require any carryover from the previous digit to next higher level.

Step 3: Normalise the sum using the Nikhilam sutra again

$$2\ \bar{1}\ \bar{6}\ 0 = 1840$$

1.4 **Example 3:**

$$\begin{array}{r} 1\ 9\ 3\ 7\ 2 \\ +\ 2\ 4\ 8\ 3\ 5 \\ \hline \end{array}$$

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Step 1: Convert all the big digits into their complements and increase previous digit by 1 in each case.

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$$2\ \bar{1}\ 4\ \bar{3}\ 2$$

$$2\ 5\ \bar{2}\ 3\ 5$$

In this case 9 and 7 in the first row are being converted to their complements. Because they are appearing as single digits, hence subtracted from 10 (according to Nikhilam). Also note that one has been added to the digits at the hundredth and at the ten thousandth place. Similarly in the second row only 8 has been converted into its compl ment because this is the only digit larger than 5 and of course the previous digit (4) is increased by one.

Step 2: Add column by column as described in the previous example

$$\begin{array}{r} 2\ \bar{1}\ 4\ \bar{3}\ 2 \\ 2\ 5\ \bar{2}\ 3\ 5 \\ \hline 4\ 4\ 2\ 0\ 7 \end{array}$$

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Step 3: This step is not required in this case because all the numbers are already in the normalised form.

1.5 **Example 4:**

$$\begin{array}{r} 3\ 8\ 6\ 5\ 5\ 9\ 5 \\ 8\ 8\ 9\ 8\ 2\ 2\ 5 \\ \hline 1\ 2\ 4\ 5\ 7\ 3\ 0 \end{array}$$

Step 1: Convert all the big digits into their complements. Now in this problem if you see little more carefully in the first column (right most) 5 and 5 are appearing. Adding 5 to 5 will provide us a two digit number and requires carrying. In order to avoid this we can convert one of the 5 into its complement. This type of situation should be watched and convert one of 5's into its compl ment (which is $\bar{5}$) will avoid carry over. The similar manipulation should also be done in column 4 (either from left or right in the case).

Thus changing necessary digits to their complements provide us

Row 1 $\quad 4\ \bar{1}\ \bar{3}\ \bar{4}\ \bar{4}\ \bar{0}\ \bar{5}$

Row 2 $\quad 1\ \bar{1}\ \bar{1}\ \bar{0}\ \bar{2}\ 2\ 2\ 5$

Row 3 $\quad 1\ 2\ 4\ 6\ \bar{3}\ 3\ 0$

Step 2: Now add column by column starting from the right most column and writing the answer below the respective column.

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$$\begin{array}{r} 4\ \bar{1}\ 3\ \bar{4}\ \bar{4}\ 0\ \bar{5} \\ 1\ \bar{1}\ \bar{1}\ 0\ \bar{2}\ 2\ 2\ 5 \\ 1\ 2\ 4\ 6\ \bar{3}\ 3\ 0 \\ \hline 1\ 4\ 0\ 1\ \bar{0}\ \bar{5}\ 5\ 0 \end{array}$$

Step 3: Normalise the resulting answer.

$$1\ 4\ 0\ 1\ 0\ \bar{5}\ 5\ 0 \quad = 14009550$$

Please note here that strictly speaking only one digit i.e. 5 is with bar here and preceding this digit 5 is 0. In order to convert it into normalised form it will be much more convenient to put a bar over zero also and treat zero as first digit and 5 as last digit. Thinking in this way will help us in applying Nikhilam with more ease and provide the positive digits directly in one line.

NOTES: 1. All these problems, their convertion to vinculum form and normalised form can be easily checked by applying the Vedic check method already discussed in Pushp 1 & 2.

2. Vedic method once again provides us with 3 different and distinct methods for even adding the numbers and fourth method would be to suitably mix up any two of them, as per convenience.

1.6 EASIEST METHOD:

The easiest, quickest and simplest method would be to use complement of bigger digits directly. Whenever we use the complement of any digit, we say 'Shudha' and put a dot on it and complement being a bar digit reduces the cumulative sum. Thus the floating sum is always within 0 to 9 only and we never need to cross 9, hence the speed and simplicity. If we have taught the children, the numbers and their complements, then whole task is a simple smiling exercise, involving no botheration. If properly practised one can easily add faster than even the calculator.

1.7 Illustration 5 : Add

$$\begin{array}{r} \dot{7}\ 4\ \dot{8} \\ \dot{2}\ 8\ \dot{9} \\ 3\ 2\ 2 \\ \dot{9}\ 7\ \dot{8} \\ \dot{2}\ 8\ \dot{4} \\ +\ \dot{4}\ 2\ \dot{9} \\ \hline 3\ 0\ 5\ 0 \end{array}$$

Step 1: Start from unit column 8 to be added to 9, instead of this we add 8 to $\bar{1}$, the complement of 9, and put a dot of Shudha on 9, (whose complement has been used). Now $8 + \bar{1} = 7$; $7 + 2 = 9$; 9 added to $\bar{2}$, the compl-

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ment of 8, and a dot of Shudha on 8; $9 + \bar{2} = 7$. Further we can use the complement of this 7, which is $\bar{3}$ and add to 4; $4 + \bar{3} = 1$, here we put a dot of Shudha on 4. Finally 1 added to complement of 9, $\bar{1} + 1 = 0$, and the 'Shudha' dot comes on 9. Therefore we get 0 at the unit place.

Step 2: Now we count the 'Shudha; dots, here 4 and add to the first digit at the top of the tenth place column $4 + 4 = 8$. Now 8 is added to complement of the next digit (8), $8 + \bar{2} = 6$, and put a Shudha dot on 8. Next $6 + 2 = 8$. Now 8 is again added to the complement of 7 ($\bar{3}$), $8 + \bar{3} = 5$, and a dot is put on 7. Next 5 is again added to complement of next digit 8 ($\bar{2}$), $5 + \bar{2} = 3$ and a dot is put on 8. Finally $3 + 2 = 5$ is the digit at the tenth place.

Step 3: We count the Shudha dots which are 3, proceed to the 3rd column and add it to complement of 7. So $3 + \bar{3} = 0$. Put a dot of Shudha on 7, and proceed further; $0 + 2 = 2$; $2 + 3 = 5$. Again say Shudha and dot on 9, $5 + \bar{1} = 4$; $4 + 2 = 6$, finally $6 + 4 = 10$ Shudha and dot on 4 and 0 is put in 3rd column.

Step 4: Count the dots of Shudha i.e., 3 and place this in 4th column. It can be checked by the use of the Sutra 'Gunit' (No.15) (Refer Pushp 1).

1.8 **Illustration 6:** Add

	7	2	4
	3	9	8
	5	3	9
	7	8	8
	2	2	2
	2	6	7	1

Step 1: $4 + \bar{2} = 2$ Shudha

$2 + \bar{1} = 1$ Shudha

$1 + 8 = 9$

$\bar{1} + 2 = 1$ Shudha

Since 9 is big , we take complement of 9 ($\bar{1}$), now Shudha dot can be

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put on either 8 or 2.Again we have a choice here.

Sttep 2: We have 3 dots of 'Shudha'

$3 + 2 = 5$;

$5 + \bar{1} = 4$

$4 + 3 = 7$

$7 + \bar{2} = 5$

$5 + 2 = 7$ is placed below the 2nd column.

Step 3: In second column we have 2 dots of Shudha which we carry; $2 + 7 = 9$ say Shudha and use

$\bar{1} + 3 = 2$

$2 + 5 = 7$

$7 + \bar{3} = 4$

$4 + 2 = 6$

Step 4: There are 2 dots of Shudha in 3rd column, hence we carry 2 to the fourth column.

1.9 Illustration 7:

Add

$$\begin{array}{r} 3\,7\,4 \\ 9\,6\,8 \\ 2\,3\,9 \\ 7\,7\,6 \\ 2\,9\,8 \\ \hline 2\,6\,5\,5 \\ \hline \end{array}$$

Let us add now from bottom to top.

Step 1: $8 + \bar{4} = 4$ (Shudha); $4 + \bar{1} = 3$ Shudha; $3 + \bar{2} = 1$ Shudha; and $1 + 4 = 5$.

Step 2: We carry 3 Shudha dots $3 + \bar{1} = 2$ (Shudha); $2 + 7 = 9$ Shudha and further we use $\bar{1}$; $\bar{1} + 3 = 2$; $2 + 6 = 8$; and $8 + \bar{3} = 5$

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Shudha.

Step 3 Shudha has been used 3 times in the previous step. Therefore $3 + 2 = 5$; $5 + \bar{3} = 2$ shudha; $2 + 2 = 4$; $4 + \bar{1} = 3$; $3 + 3 = 6$.

Step 4: The 2 dots of Shudha are placed in 4th column as 2.

1.10 DISCUSSIONS

1. These three illustrations should be sufficient to explain the rather simple, easy and superfast Vedic Method of addition, which uses two vedic concepts of complements of numbers obtained by Nikhilam sutra and the Shudha upsutra.

2. Since we have hardly worked this way, we shall naturally have the intellectual inertia in adopting it.

3. The practise of addition the Vedic way is the sure solution and we can work we speed andf accuracy, as we never cross 9 and are always playing within 0 -9 range, which is far more easier, simple and hence has lesser chances of error.

PROBLEMS:

Q.1 Add the following:

i) 2854 + 2557 + 5411

ii) 454632 + 245416 + 700048

iii) 25464 + $\overline{15465}$ + 10009

Q.2 Perform the following operations:

i) $4\ \bar{2}\ \bar{6} + \bar{3}\ 4\ 7 + 2\ 4\ 5 + \bar{5}\ 6\ 7 + 4\ \bar{7}\ \bar{3}$

ii) $3\ \bar{2}\ \bar{5} + \bar{3}\ \bar{4}\ 6 + 5\ 0\ \bar{1} + \bar{3}\ 1\ 1 + 5\ 6\ \bar{6} + \bar{4}\ 7\ 3$

iii) $1\ 6\ \bar{6} + \bar{5}\ 6\ 6 + 2\ 5\ 5 + 8\ \bar{1}\ \bar{1} + 4\ 5\ \bar{5} + \bar{3}\ 5\ 6$

Q.3 Check the above results by

(a) Vedic Method

(b) by converting all to vinculum numbers and then simplify.

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CHAPTER 2

SUBTRACTION USING VINCULUM

2.1 INTRODUCTION

We have already seen in Chapter 5 of Pushp 1 how to use 'Shudha' subsutra for subtraction.Shudha method is quite simple in comparison to present technique but still we had to make a series of additions and subtractions.More simpler and natural approach for carrying out subtraction will be the use of Vinculum. We can think of atleast two possibilities in which we can make use of vinculum and hence have the following two methods.

2.2 CONVERTING TO BAR FORM

Here all the digits to be subtracted are straight written with bar. This technique will essentially convert the subtraction process into an addition process and thus no subtraction is required at all.

Let us demonstrate it with the help of an example.

2.2.1 Examples:

$$\begin{array}{rr} & 3\ 2\ 4\ 7 \\ - & 1\ 8\ 2\ 8 \\ \hline & \end{array}$$

The following steps are involved:

Step 1: Because we have to subtract 1828 from 3247. That actually means the entire number 1828 is a negative number and instead of writing the number as -1828 we may write it as $\bar{1}\ \bar{8}\ \bar{2}\ \bar{8}$.

$$\begin{array}{r} 3\ 2\ 4\ 7 \\ \bar{1}\ \bar{8}\ \bar{2}\ \bar{8} \\ \hline \end{array}$$

Step 2: Once we have written the number in the form $\bar{1}\ \bar{8}\ \bar{2}\ \bar{8}$ that means now we can add this bar number to the number 3 2 4 7.

$$\begin{array}{rr} & 3\ 2\ 4\ 7 \\ + & \bar{1}\ \bar{8}\ \bar{2}\ \bar{8} \\ \hline & 2\ \bar{6}\ 2\ \bar{1} \end{array}$$

$7 + \bar{8} = \bar{1}$; $2 + \bar{8} = \bar{6}$ etc.

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Step 2: Rest of the problem is nothing but to normalise this number $2\ \bar{6}\ 2\ \bar{1}$ using Nikhilam sutra. Thus $2\ \bar{6}\ 2\ \bar{1} = 1419$ (Refer Pushp 1 & 2).

2.2.2 Example 2:

$$\begin{array}{r} 4\ 3\ 5\ 1\ 0 \\ -\ 3\ 7\ 8\ 2\ 9 \end{array}$$

Step 1: Change the number to be subtracted in to the bar form.

$$\begin{array}{r} 4\ 3\ 5\ 1\ 0 \\ +\ \bar{3}\ \bar{7}\ \bar{8}\ \bar{2}\ \bar{9} \end{array}$$

Step 2: Add the two numbers:

$$\begin{array}{r} 4\ 3\ 5\ 1\ 0 \\ +\ \bar{3}\ \bar{7}\ \bar{8}\ \bar{2}\ \bar{9} \\ \hline 1\ \bar{4}\ \bar{3}\ \bar{1}\ \bar{9} \end{array}$$

Step 3: Normalise the result using Nikhilam sutra.

Thus $1\ \bar{4}\ \bar{3}\ \bar{1}\ \bar{9} = 5681$

1.2.3 Example 3: In this example we will take a case where bigger of the number is to be subtracted.

$$\begin{array}{r} 3\ 5\ 5 \\ -\ 4\ 6\ 4 \\ \hline \end{array}$$

Step 1: Change the number to be subtracted in the bar form.

$$\begin{array}{r} 3\ 5\ 5 \\ \bar{4}\ \bar{6}\ \bar{4} \\ \hline \end{array}$$

Step 2: Add the two numbers.

$$\begin{array}{r} 3\ 5\ 5 \\ +\ \bar{4}\ \bar{6}\ \bar{4} \\ \hline \bar{1}\ \bar{1}\ 1 \end{array}$$

Step 3: Normalise the result using Nikhilam sutra.

$\bar{1}\ \bar{1}\ 1 = \bar{1}\ 8\ 9\ 1$

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Because the first digit from LHS is a bar digit it is obvious that the resulting number will be a negative number and thus using once again Nikhilam we can write this number as -109 (This time we have applied the Nikhilam only over the positive digits)

Instead of doing this we could have directly written $\bar{1}\ \bar{1}\ 1$ as -109 just by Vilokanam or mere observation because the number $\bar{1}\ \bar{1}\ 1$ is nothing but -100 -10 + 1 =-109· or in other words converting the positive digits also into the bar digits and then taking out bar as a minus sign.

2.3 DIRECT METHOD

In the second method we may subtract the lower number directly from the upper number. If the lower digit is bigger than the respective upper digit than we write the answer in terms of the bar and if the lower digit is smaller than the upper digit, we carry out usual subtraction.

Let us demonstrate the procedure with one example.

2.3.1 Example 4:

$$\begin{array}{r} 4\ 3\ 5\ 8 \\ -\ 3\ 4\ 2\ 9 \\ \hline \end{array}$$

Step 1: Subtract the lower digit of first column i.e. 9 from 8 in this case. Because 9 is larger than 8 hence we get result as $\bar{1}$

$$\begin{array}{r} 4\ 3\ 5\ 8 \\ -\ 3\ 4\ 2\ 9 \\ \hline \\ \bar{1} \end{array}$$

Step 2: Now subtract the digit of the second column, as 2 is smaller than 5 the resulting digit will be a positive one i.e., 5 - 2 = 3.

$$\begin{array}{r} 4\ 3\ 5\ 8 \\ -\ 3\ 4\ 2\ 9 \\ \hline \\ 3\ \bar{1} \end{array}$$

Step 3: Repeat the above process for remaining columns and we get

$$\begin{array}{r} 4\ 3\ 5\ 8 \\ -\ 3\ 4\ 2\ 9 \\ \hline \\ 1\ \bar{1}\ 3\ \bar{1} \end{array}$$

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Step 4 : Normalise the number using Nikhilam sutra.

$$1\ \bar{1}\ 3\ \bar{1} \quad = \quad 929$$

NOTE: It is interesting to note here that in the above two methods we can start either from right to left or from left to right. This is possible because Shudhikaran is not required for any digit. However, in some rare cases we may require Shudhikaran of some digits and then in that case calculations should be done only from right to left.

PROBLEMS:

Q.1 Subtract the following:

i) $4\ \bar{2}\ \bar{4}\ \bar{8} - 2\ \bar{1}\ \bar{2}\ \bar{9}$

ii) $3\ 5\ \bar{8}\ \bar{5} - 5\ 3\ \bar{9}\ \bar{2}$

iii) $3\ 4\ 7\ 2\ 3 - 1\ 4\ 1\ 3\ 4$

iv) $8\ \bar{9}\ \bar{9}\ \bar{9}\ \bar{5}\ \bar{2} - 4\ 5\ 4\ 6\ 4\ \bar{8}$

v) $4\ \bar{2}\ \bar{4}\ \bar{7} - 8\ 9\ 9\ 9$

Q.2 Convert all the numbers of Q.1 into the normal form and then perform the subtraction by all possible alternative Vedic methods.

Q.3 Check the results obtained using the simple mental Vedic check method.

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CHAPTER 3

DIFFERENT BASE SYSTEM

3.1 INTRODUCTION

So far we have seen the use of only a single base or a sub-base for solving any multiplication problem. Same has been the case for the problems requiring the use of sub-base. In this approach one of the deviations becomes very large if one of the numbers is far away from the base, thereby requiring the product of larger digits. The question may be raised as to whether the 'Nikhilam' method can be used with different base for different numbers of one product. The answer to this is**YES**. As a matter of fact, the 'Nikhilam' method becomes much more efficient, versatile and convenient through this extension.Therefore, for multiplying two numbers which are far from each other, we can choose different base.

3.2 EXAMPLES :

Illustration 1: Multiply 109 x 12.

We choose base for 109 as 100 and base for 12 as 10.

The deviation for each number from the respective base may now be written in a normal 'Vedic style'.

```
109 + 09
 12 + 2
--------
```

Step 1: The R.H.S. is obtained as usual by multiplying two deviations. Therefore in present case, 2 x 9 =18.

Step 2: For obtaining the L.H.S. portion of the product we have to use the ratio of the two bases selected. In this case 100/10 = 10 is the required ratio.

The only modification in the process of obtaining the L.H.S. is that the number of the deviation,having the lower base, shall be multiplied by this ratio (of the two selected bases) while adding to the second number or its deviation, as the case may be.

L.H.S. = 109 + 2 x 10 = 129

Alternatively L.H.S. = 12 x 10 + 9 =129 since the first number's base is 10 times the base of the second number hence we multiply the deviation of the second number (which is 2) by the base ratio (10) and add it to the bigger number (109).Alternatively we can multiply the smaller number (12) by the base ratio (10) and then add it to the deviation of the larger number.

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```
109 + 09          Base      =  100
 12 + 2           Base      =   10
--------                          100
129 / 18          Base ratio =    ---   =   10
                                   10
```

Step 3: We have to note that in this process the number of digits to be kept on the R.H.S. shall be equal to the number of zeros in the<u>smaller base</u>.

In the present example, the smaller base is 10. Therefore only one digit is permissible and the excess digit is carried to the left, as usual.

129 / 8 = 130 /8 = 1308

Illustration 2: Multiply 1009 x 13.

```
1009 + 009          Base   = 1000
  13 +   3          Base   =   10
----------
1309 /   7
    / 2
```

= 1309/7 R.H.S. = 9 x 3 = 27
2/ L.H.S. = 1009 + 3 x 100 = 1009 + 300 = 1309

= 13117 or L.H.S. = 13 x 100 + 009 = 1309

NOTES:

1. We get the same L.H.S. by following any of the two alternatives, discussed in the previous example.

2. The same R.H.S. can also be obtained even by following the extension of the third and fourth alternatives already discussed in the basic chapter of 'Nikhilam' sutra.Refer Pushp 1)

(a) If we add the deviations of the two numbers (considering the suitable weightage in the form of the base ratio) to the larger base we get the L.H.S.

In this case (3 x 100 + 9) + 1000 = 1309

(b) The fourth alternative way of obtaining the L.H.S. is to add the two numbers, of course with due weightage, and then subtract the larger base from this addition. Here 1009 + 13 x 100 = 2309.

Therefore L.H.S. = 2309 - 1000 = 1309.

So we observed that the wonderful unique and useful quality of 'Vedic' system,of providing easy and multiple alternative methods, for getting at the part of the answer is available even in this extension of the 'Nikhilam' method. It shall not be out of place to mention again that we can use any of these alternative methods to

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get at the answer and check our computations by computing the result following any of the remaining alternatives.

COMMENT: This extension of the 'Nikhilam' method further simplify the already simple 'Nikhilam' method, through the use of different bases.The following solutions illustrate this observation.

1. Base 1000

1009 + 009
13 - 987
---

22 / $\overline{883}$ = 14 / $\overline{883}$ = 13117

$\overline{8}$ /

2. Base = 10 is the alternative choice.

1009 + 999
13 + 3

1012 / 7 = 13117
/299

Illustration 3: The extension of the 'Nikhilam' method is also applicable for all types of cases covered by the basic 'Nikhilam' method. Let us take another example, requiring the use of two bases but one number is larger and the other smaller than the selected bases.

Multiply 1008 x 96.

1008 + 008 Base =1000
96 - 4 Base = 100
--- Base ratio =10

968 / $\overline{32}$
= 96768 R.H.S. = $\overline{32}$

L.H.S. = 96 x 10 + 8 = 968

Illustration 4: Multiply 10007 x 989.

10007 + 0007 Base = 10000
989 - 011 Base = 1000

9897 / $\overline{077}$ . . Base ratio = $\frac{10000}{1000}$ = 10

= 9896923 R.H.S. = $\overline{77}$

L.H.S. = 989 x 10 + 7 = 9897

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or L.H.S. = 10007 - 11 x 10

= 10007 - 110

= 9897

Illustration 5: Multiply 89 x 9988.

9988 - 0012 Base = 10000

89 - 11 Base = 100

8888 / 32 . . Base ratio = 100

/ 1

R.H.S = -12 x -11 =132

= 888932

L.H.S. = 89 x 100 - 12

= 8900 - 12 = 8888

or L.H.S. = -11 x 100+ 9988

= 9988 - 1100 = 8888

PROBLEMS

Q.1 Multiply the following numbers using appropriate bases, using two different bases.

i) 989 x 19 ii) 1047 x 92 iii) 10015 x 981

iv) 99898 x 989 v) 112 x 14 vi) 100018 x 998

vii) 99889 x 112 viii) 989 x 11002 ix) 99 x 9989

x) 123 x 10103

Q.2 Check all the results obtained in Q.1. using the vedic multiplication check method based on Vedic Sutra number 14.

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CHAPTER 4

DIFFERENT SUB-BASES

4.1 INTRODUCTION

In the previous Pushp 1 & 2 we have seen how the 'Nikhilam' method can be used for solving different types of problems.We have also studied how the 'Nikhilam' method can be used for multiplication of even more than two numbers in one line which is possible only in the Vedic Method (Pushp 2). In the chapter just studied we have further seen how the 'Nikhilam' method becomes all the more versatile by the use of different bases.The 'Nikhilam' method becomes extremely useful if we take different sub-bases. In this group two types of situations may arise. In the first type, one of the numbers may be close to a base and the second number is closer to a suitable sub-base. In the second type, both of the numbers may be closer to two suitable sub-bases.

4.2 ILLUSTRATIONS: TYPE 1

Let us take a couple of examples of the first type to see the application of the 'Nikhilam' sutra.

4.2.1 Illustration 1: Multiply 109 by 21.

The steps are essentially the same as outlined in the previous chapter. The following solution shall explain the simple process. In this example, the first number 109 is closer to the base 100 and the second number 21 is closer to the sub-base 20 (10 x 2).The L.H.S. can be obtained by two alternative approaches using the base ratio as already explained.

Base		Sub-base
100		
10		10 x 2 = 20
109 + 9		Base ratio = 100/10 = 10
21 + 1		R.H.S. = 9 x 1 = 9
-------		L.H.S. = 2 x 109 + 1 x 10= 228
228 / 9		
	or	L.H.S. = 9 x 2 + 21 x 10 = 228

= 2289

Illustration 2: Multiply 1007 x 309.

Base		Sub-base
1000		
100		3 x 100 =300
1007 + 007		Base ratio = 1000/100 =10
309 + 09		R.H.S. = 7 x 9 = 63
----------		L.H.S. = 1007 x 3 + 9 x 10
3111 / 63		= 3021 + 90 = 3111
	or	L.H.S. = 309 x 10 + 7 x 3
		= 3090 + 21 =3111

= 131163

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In the above illustrations we observe that the additional effort involved in handling even a number closer to a suitable sub-base is very small, but however the efficiency of the system increases tremendously.

4.3 ILLUSTRATIONS TYPE 2:

As already mentioned it is possible even to solve the multiplication problems of numbers closer to two suitable different sub-bases. The 'Nikhilam' method becomes all the more handy when we choose different sub-bases. Through this approach rather a broad class of multiplication problems can be very conveniently solved by using suitable sub-bases and drastically reducing the size of the deviations.The simple step by step process shall be clear from the following examples.We multiply the different sub-bases by suitable numbers such that the L.C.M. for both the sub-bases will be equal, in computing the left hand side of the answer.

4.3.1 Illustration 1: Multiply 129 x 31.

The ordinary 'Nikhilam' approach shall apparently be unsuitable, due to large deviations.

Base	Sub-base
10	12 x 10
10	3 x 10
129 + 9	L.H.S. = 9 x 3 + 31 x 12 = 399
31 + 1	
399 / 9	Alternatively, 129x3 + 1x12 = 399

Ans. 3999

Here 31 is multiplied by 12 and 9 is multiplied by 3 because if first sub-base is multiplied by 3 and second sub-base by 12, both the multiplications yield equal values. Therefore, 120 x 3 = 360 and 30 x 12 = 360

4.3.2 Illustration 2. Multiply 59 x 37.

Base	Sub-base	
10	4 x 10 = 40	
10	3 x 10 = 30	
49 + 9	L.H.S.. = 9 x 3 + 37 x 4 = 27 + 148	
37 + 7	= 175	
175/ 3	or L.H.S. = 7 x 4 + 49 x 3 = 28 + 147	
6		
= 1813	= 175	

Ans. 1813

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4.3.3 Illustration 3: Multiply 311 x 721.

Base	Sub -base
100	3 x 100 =300
100	7 x 100=700
311 + 11	L.H.S. = 721 x 3 + 11 x 7
721 + 21	= 2163 + 77 = 2240
2240/ 31	or L.H.S. = 311 x 7 + 21 x 3
2	= 2177 + 63 = 2240
= 224231	

Ans. 224231

4.3.4 Illustration 4: Multiply 259 x 512.

Base	Sub-base
1000	$\frac{1}{4} \times 1000 = 250$
1000	$\frac{1}{2} \times 1000 = 500$
259 + 9	L.H.S. = $259 \times \frac{1}{2} + 12 \times \frac{1}{4}$
512 + 12	
1	$= 129\frac{1}{2} + 3 = 132\frac{1}{2}$
$132\frac{1}{2}$ /108	
= 132608	or L.H.S. = $512 \times \frac{1}{4} + 9 \times \frac{1}{2}$
	$= 128 + 4\frac{1}{2} = 132\frac{1}{2}$

$\frac{1}{2}$ on L.H.S. is $\frac{1000}{2}$ = 500 on R.H.S. as usual. Once again here

$250 \times \frac{1}{2} = 125$ which is the same as $500 \times \frac{1}{4} = 125$.

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4.3.5 llustration 5 Multiply 307 x 4015.

Base	Sub-base
100	3 x 100 =300
1000	4 x 1000=4000

307 + 07
4015 + 15

12325/ 05
1

= 1232605

Base ratio =1000/100 = 10
L.H.S. = 307 x 40 +15 x3
= 12280 + 45 = 12325

or L.H.S. = 4015 x 3 + 7 x 40
= 12045 +280 = 12325

Ans. 1232605.

4.3.6 Illustration 7: Multiply 387 x 2112.

Base	Sub-base
100	4 x 100=400
1000	2 x 1000=2000

387 - 13
2012 + 12

7788/ $\overline{56}$
1

= 7787 / $\overline{56}$
= 7786/100-56 = 778644

Base ratio = 1000/100 = 10
L.H.S. = 387 x 20 + 12 x 4
= 7740 + 48 = 7788

or L.H.S. = 2012 x 4 - 13x20
= 8048-260 = 7788

Ans. 778644

4.3.7 Illustration 8: Multiply 1142 by 81.

Base	Sub-base
100	11 x 100= 1100
100	8 x 10 =80

1142 + 42
81 + 1

9246/ 2
4

= 92502

Base ratio = 100/10 = 10
L.H.S.= 42 x 8 + 81 x 110 =9246

or L.H.S.= 1142 x 8 + 1 x110
= 9246

81 is multiplied by 110 and 42 by 8 and second if first sub-base is multiplied by 8 and second sub-base by 110, then both the sub-bases will be equal (8800).

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4.4 DISUCSSIONS

1. We observe that the range of the problems which can be easily solved by using the 'Nikhilam' method increases tremendously by using the simple extensions of the 'Nikhilam' method.

2. The computations involved in this are most of the times very simple. Therefore, bulk of these, after a little practice, can be done mentally thereby reducing the computation time.

3. Further the L.H.S. of the answer in the vedic method is obtainable by two very convenient alternatives.We can obtain the L.H.S. by one and check the correctness of our computations by recomputing the L.H.S. by the second approach.

4. At the top of it the superfast, almost magic speed method of checking the correctness of the multiplication can be used to confirm the results.

5. It is to be mentioned that the R.H.S. shall always contain the number of digits equal to the number of zeros in the smaller of the two bases selected.

PROBLEMS

Q.1 Multiply the following using different sub- bases:

i) 36 x 152	ii) 195 x 295	iii) 281 x 589
iv) 645 x 982	v) 1321 x 821	vi) 11856 x 897
vii) 4025 x 75	viii) 12456 x 1259	ix) 307 x 1504
x) 6056 x 984	xi) 2034 x 1015	

Q.2 Check the results of Q.1 using the Vedic check method based on the Vedic Sutra number 14.

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CHAPTER 5

THREE NUMBER MULTIPLICATION - SUB BASES

5.1 INTRODUCTION

The extension of the 'Nikhilam' sutra for multiplication of three, four, five or even more numbers in one line has been briefly illustrated in the previous Pushp 2. The question naturally arises whether it is possible to multiply even the numbers which are not closer to the base but are closer to some suitable sub-base. Interestingly the answer is yes. The superfast 'Vedic Nikhilam method' can be easily extended to handle numbers closer to the sub-base.

5.2 THE METHODS

The process outlined in the sub-base chapter is directly applicable to the sub-base system also, even for the multiplication of three or more numbers. The following examples shall illustrate the process. The only difference is in the application of the corollary 'Anurupena'. In the case of three number multiplication we modify the central part of the product by multiplying it with the base multiple and the left hand portion of the result is modified with the square of this multiple.

5.3 EXAMPLES:

Illustration 1: 48 x 54 x 45.

Step 1: Base 10
Sub-base 5 x 10 = 50
. .
. . The base multiple = 5

Step 2: We find the deviations of each number from the sub-base selected as usual.

```
48 - 2
54 + 4
45 - 5
------
 /  /
```

Step 3: The product as we already know shall consist of three parts and therefore we draw two slashes.

Step 4: The right hand part of the product is simply the algebraic product of the three deviations. We do not need to modify this part of the answer, as before.

Step 5: The raw central portion of the product is obtained as the summation of the three products, taking two deviations at a time. This is written within the two slashes.

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Step 6: The raw left hand portion of the product can once again be obtained by the algebraic addition of any of the three numbers with the deviations of the other two numbers from the sub-base.

In the present example, we find

R.H.S. = (-5) x (4) x (-2) = 40

Central portion = (-5 x 4) + (-5 x -2) + (4 x -2)

= - 20 +10 - 8 = -18

L.H.S. = (48 -5 +4) = 47

Base = 10, Subbase = 5 x 10 = 50

48 - 2
54 + 4
45 - 5
---

47 / $\overline{18}$ /40

It goes without saying that the L.H.S. can also be obtained by using the second or third number for addition with the deviations of the other two numbers.

Step 7: The final central portion of the product is obtained by invoking the 'Anurupena' sub-sutra and thereby multiplying the raw result by the base-multiple or sub-multiple, as the case may be.

In the present case the final central part of the product is equal to 5 x $\overline{18}$ = $\overline{90}$.

Step 8: The final L.H.S. of the product is obtained by multiplying the raw L.H.S. by the square of the base multiple or sub-multiple.

In this example therefore L.H.S.= 47 x 5 x 5 = 1175

Note: We have to mark this point that the carry over of the excess digit of any portion of the answer shall not be done till the final form of each part of the product has been obtained.

In the present example, since our base is 10, only one digit is finally permissible in each part of the product and the excess digits of the finalised products are carried over as usual. Therefore

1175 /0 /0
$\overline{9}$/ 4/

$117\overline{4}$ 4 0 = 116640

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Alternative Solution:

The previous example can alternatively be solved by considering a base of 100 (instead of 10) and sub-base of 50.

$$\text{Base} = 100 \quad \text{Sub-base} = \frac{100}{2} = 50$$

$$\therefore \text{The base sub multiple} = \frac{1}{2}$$

48 - 2
54 + 4
45 - 5

R.H.S. = 40

Central portion = $\overline{18}$

L.H.S. = 47

47 /$\overline{18}$/ 40

We get the raw values of each portion by following the steps from 1 to 6 as before. The last two steps get modified, as the base submultiple, in this case is 1/2.

$$\text{Therefore, the final central part} = \frac{1}{2} \times \overline{18} = \overline{09}.$$

$$\text{Final L.H.S.} = 47 \times \frac{1}{2} \times \frac{1}{2} = 11\frac{3}{4}.$$

Further, since the base is 100, we require two digits in the last two portions of the resulting product.

$$\text{Therefore, product} = 11\frac{3}{4} \,/\, \overline{09} \,/ 40$$

$\frac{3}{4}$ at this place = 7500

$$\therefore \quad 11 \,/\overline{09}\, / \,40 = 11/66/40$$
$$\quad\quad\quad 75$$

Once again we get the same answer.

Illustration 2: Multiply 253 x 248 x 252.

$$\text{Base} = 1000, \quad \text{Sub-base} = 250 = \frac{1000}{4}$$

$$\therefore \text{Sub-multiple} = 1/4$$

253 + 3
248 - 2
252 + 2

L.H.S. = 253

Central part = $\overline{4}$

R.H.S. = $\overline{12}$

253 /$\overline{004}$ /$\overline{012}$

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$$\text{Final central part} = \frac{1}{4} \times \overline{004} = \overline{001}$$

$$\text{Final L.H.S.} = \frac{1}{4} \times \frac{1}{4} \times 253 = 15\frac{13}{16}.$$

$$\frac{13}{16} \text{ at this place} = \frac{13}{16} \times 1000000$$

$$= 812500$$

$\therefore$ Product $= 15/\overline{001}/\overline{012}$
$\quad\quad\quad

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Q.3 Multiply the following using sub-bases.

i) 253 x 247 x 252 ii) 25 x 28 x 19

iii) 547 x 553 x 558 iv) 249 x 242 x 256

v) 53 x 46 x 57

Q.4 Check all the results by using Vedic check method.

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CHAPTER 6

SQUARING URDHVA AND YAVADUNAM SUTRAS

6.1 URDHVA METHOD:

6.1.1 INTRODUCTION:

We have seen that how the Urdhva method is used for the multiplication of two numbers or more than two numbers. We can also use the Urdhva method for squaring. The following examples described the procedure :

6.1.2 ILLUSTRATION 1: Square the number 39.

Step 1: First we write the number as

```
  39
  39
 ----
```

Step 2: We draw two slashes for three parts of the result.

```
  3 9
  3 9
------------
R  /R  /R
 3   2   1
```

STAGE I

Step 3: Multiply the digits of Ist column, vertically and get R_1.

$$R_1 = 9 \times 9 = 81$$

Step 4: Now multiply the digits of Ist and second column crosswise, and get R_2 as

$$R_2 = 3 \times 9 + 3 \times 9 = 27 + 27 = 54$$

$$\text{OR } R_2 = 2 \times 3 \times 9 = 54$$

STAGE II

Step 5: Further leaving the Ist column, we multiply the digits of 2nd column only and get R_3.

$$R_3 = 3 \times 3 = 9.$$

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Step 6: Now we add the carry digits and get result

```
      39
      39
   -----------
   9 /  4/  1        = 1521
      5.  8
```

So $39^2 = 1521$

6.1.3 ILLUSTRATION 2: Solve 77^2.

Step 1: We write as

```
   7 7
   7 7
   ----
```

Step 2: Draw slashes

```
   7 7
   7 7
------------
 R  /R  /R
  3   2   1
------------
```

STAGE I

Step 3: $R_1 = 7 \times 7 = 49$

Step 4: $R_2 = 7 \times 7 + 7 \times 7 = 49 + 49 = 98$

STAGE II

Step 5: $R_3 = 7 \times 7 = 49$

Step 6: Add the carry digits to get the result.

```
      7 7
      7 7
   -----------
   49 / 98/ 49     =  49 /  8 /  9    =  5929
                         9     4
```

6.1.4 ILLUSTRATION 3 Solve 352^2.

The method of finding the square of three digit number is same as described in the above two illustrations (Refer Pushp-1 Chapter-10).

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Step 1: We write as

```
3 5 2
3 5 2
------
```

Step 2: Draw slashes

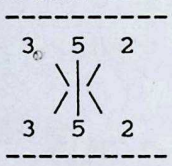
```
3 5 2
3 5 2
-----------
/ / / /
```

STAGE I

Step 3: Multiply vertically the digits of Ist column, we get R_1.

$$R_1 = 2 \times 2 = 4$$

Step 4: Now multiply the digits of 1st and 2nd column crosswise we get R_2.



$$R_2 = 5 \times 2 + 2 \times 5 = 10 + 10 = 20$$

OR $R_2 = 2 (5 \times 2) = 20$

Step 5: Further multiply the digits of Ist and 3rd column crosswise and 2nd column vertically.

$$R_3 = 3 \times 2 + 3 \times 2 + 5 \times 5 = 6 + 6 + 25 = 37$$

STAGE II Elimination:

Step 6: Now we leave the first column and multiply the digits of 2nd and 3rd column

```
3 5
 x
3 5
-----
```

$$R_4 = 3 \times 5 + 3 \times 5 = 15 + 15 = 30$$

Step 7: Further we multiply the digits of 3rd column vertically we get R_5 as

$$R_5 = 3 \times 3 = 9$$

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Step 8: Now add the carry digits to get the result.

```
    3 5 2
    3 5 2
  ------------
  9/ 0/ 7/ 0/4        = 123904
    3  3  2
```

6.1.5 ILLUSTRATION 4 Solve 772^2.

```
    7 7 2
    7 7 2
  ------------
  49/ 8/ 7/ 8/4       =  595984
     9  7  2
```

6.1.6 ILLUSTRATION 5 Solve 2306^2.

```
      2 3 0 6
      2 3 0 6
  -----------------
  4/ 2  9/4/ 6/0 /6    =  5317636
    1     2  3    3
  -----------------
```

NOTE:

After some practice the carry digit can be directly added in the next level number.

6.2 **LEFT TO RIGHT:**

We can also proceed with the computations from left hand side and the sequence of steps will be simply reversed. But generally we would need to do this in two steps as the carry over digits shall be added in the second step. However, if we adopt the unique carryover approach, of Vedic maths system, to the lower side and also use the suitable alternative form of results at each stage, we can get the total result in a single step. Here we keep the 10th level digit at the same place and shift the unit digit to the R.H.S. which then assumes 10 times its value.

6.2.1 Illustration 7: Square 23.

(i) Squaring from left to right (Ref. Pushp -2, Chap. 8)

```
    2 3
    2 3
  --------
  4 2 9   =  529
   1
  --------
```


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STAGE I:

Step 1: $RL_1 = 2*2 = 4$

Step 2: $RL_2 = 2*3 + 2*3 = 12$

STAGE II: We start the elimination from L.H.S.

Step 3: $RL_3 = 3*3 = 9$

Step 4: Add the carryover digits.

(ii) Square from left to right in one line.

```
    2 3
    2 3
 ----------
 0/ 5/ 2 9     =   529
   4   2
 ----------
```

STAGE I:

Step 1: $RL_1 = 2*2 = 04$ we retain 0 at this level and shift the unit place digit (Here 4) to the R.H.S. which naturally gets its value ten times increased (4*10 = 40).

Step 2: $RL_2 = 2*3 + 2*3 = 12$. To this we add the carried down number (from L.H.S.) 40. Therefore 12 + 40 = 52 and out of this number we keep the tenth place digit (5) at this level and once again shift the unit place digit (2) to R.H.S. which again gets its value increased ten times (2*10 = 20).

STAGE II:

Since we have already completed the operation of all columns (here only 2) with the left most column. We can start the second stage of elimination from L.H.S. and simplification of the remaining columns.

Step 3: $RL_3 = 3*3 = 9$. To this we add the carried down number (from L.H.S.) 20 + 9 = 29. Since this is the last step, therefore the number 29 is written in the right most space.

NOTES:

(1) Proceeding like this we have obtained the results in one line.

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(2) Since all the digits are small, there was no need to use the vinculum form of the result obtained in different steps. But when we are handling bigger digits we shall find in more conveient and efficient to use the vinculum form of the intermediate results as has been illustrated in the next example.

(3) We should note that in this method we shall always have two digits in the right hand most space. Even if we get a single digit value in last step it shall be written with a zero prefixed on its L.H.S.

6.2.2 Illustration 8: Square 39.

(i) Squaring from left to right (Ref. Pushp-2, Chap. 8)

```
   3 9
   3 9
 ---------
  9 4 1     =   1521
    5 8
 --------
```

STAGE i:

Step 1: RL(1) = 3*3 = 9

Step 2: RL(2) = 3*9 + 3*9 = 54

STAGE II: We start the elimination from L.H.S.

Step 3: RL(3) = 9 x 9 = 81

Step 4: Add the carry digits.

(ii) Squaring from left to right in one line.

STAGE I:

Step 1: RL(1) = 3*3 = 9

but it will be more convenient to write this in the vinculum form.

```
   3 9
   3 9
 ----------
 1/_5/_2 1
   1  6      =  1521
```

So $9 = 1\,\bar{1}$, we retain 1 at this level and shift the unit place digit ($\bar{1}$) to the R.H.S. which gets its value increasaed 10 times. Hence $\bar{1} \times 10 = \bar{1}0$. As otherwise the cumulative total shall consists of three digits and the hundredth

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place digit shall spil back to this level. (90 + 27+27=144).

Step 2: RL $_2$ = 3*9 + 3*9 = 54. To this we add the carried down number from L.H.S. (-10). Therefore 54 +(-10) = 44. Once again if we carry down the unit place digit (4) we shall end up in three digits at the next step, therefore it shall be convenient to use the vinculum form of this value i.e. 44 = $5\bar{6}$ we retain the tenth level digit(5) and shift the unit place digit ($\bar{6}$) to the R.H.S. which naturally gets its value increased by 10 times. So $\bar{6}$ * 10 = $\overline{60}$.

STAGE II:

Step 3: RL $_3$ = 9*9 = 81. To this we add the carried down number from L.H.S. (Here $\overline{60}$). So 81 + (-60) = 21 and since it is the last simplification we keep two digits at the right most space.

6.3 YAVADUNAM METHOD

There is yet another method of squaring.This upsutra reads as follows Yavadunam Tavadurikrtya Vargarca Yojayet(यावदूनं तावदूनीकृत्य वर्गं च योजयेत् *No.7*) which means whatever the extent of its deficiency lessen it to the same extent and set up the square of the deficiency (Refer Pushp 1 Chapter 9).

6.3.1 **ILLUSTRATION 9** Calculate the square of the number 98.

Step 1: First we select the base nearest to the number. Here 100 is the base.

Step 2: We divide the result in two parts by drawing a slash

98^2 = L.H.S/ R.H.S.

Step 3: The deficiency of the number from the base is (100 - 98 = 02) = 02.

Step 4: Now we subtract this deficiency from the number to get the L.H.S. as 98 - 02 = 96.

So 98^2 = 96/R.H.S.

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Step 5: For R.H.S. result we take the square of the deficiency.So the square of 02 is 04. Hence the result is $98^2 = 96/04 = 9604$. As there are 2 zeroes in the base (100) we must have 2 digits on R.H.S.

6.3.2 ILLUSTRATION 10 Calculate 996^2.

L.H.S. $996 - 004 = 992$

R.H.S $(004)^2 = 016$

Hence $996^2 = 992/016 = 992016$

6.3.3 ILLUSTRATION 11 Calculate 1013^2.

Base is 1000 and deviation is +13

$1013^2 = 1026/(013)^2 = 1026/169 = 1026169$

Note: Here the number is bigger than the base so we add the deficiency to the number.

6.3.4 ILLUSTRATION 12 Calculate 100019^2.

The base is 100000.

$100019^2 = (100019 + 00019)/(00019)^2 = 100038/00361$

$= 10003800361$

For squaring 19 we can use the Urdhva method mentally or the same method using the base 10.

(i)
```
   19
   19
-------
  8  1
1/1 /8
```
$= 361$

OR

(ii) $19^2 = 19 + 09/(09)^2$

$= 28/8^1$

Base is 10 so 8 is carried to LHS

$= 28 + 8/1 = 36/1 = 361$

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6.3.5 ILLUSTRATION 13 Calculate 1000098^2.

The base is 1000000.

Hence $1000098^2 = (1000098 + 000098)/(000098)^2$

$= 1000196 / (000098)^2$

But $(98)^2 = 98 - 2/(02)^2 = 9604$

So $1000098^2 = 100196/009604$

$= 100196009604$

6.3.6 ILLUSTRATION 14 Calculate 1037^2.

The base is 1000.

$\therefore 1037^2 = (1037 + 037)/(037)^2$

$= 1074/ 0\ 3\ 7$
$\quad\quad 0\ 3\ 7$
$\quad\quad ----------$
$\quad\quad 0\ 9/\ 2/\ 9 \quad = 1369$
$\quad\quad\quad /4\ /4$
$\quad\quad ----------$

$= 1074/1369$

Here R.H.S. is greater than the base so we carry the excess digit 1 to the L.H.S. as

$= 1074/ 369$
$\quad\quad \sim 1$
$= 1074+1/369$

$= 1075/369$

$= 1075369$

6.4 ANURUPENA SUB-SUTRA

The application of Yavadunam can be further extended to numbers away from the base but closer to any subbase by using the Anurupena sub-sutra. Once again only the L.H.S. is modified proportionately and R.H.S. stays as it is. The deviations are taken only from the subbase (Refer Chapter 8, Pushp 1).

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6.4.1 **ILLUSTRATION 15** Square 29.

Subbase = 30 = 3x(10)

L.H.S. =3(29-1) = 3 x 28 = 84

R.H.S. = $(-1)^2$ = 1 as base has 1 zero

29^2 = 841

6.4.2 **FURTHER ILLUSTRATIONS 16**

$42^2 = 4(42 + 2)/2^2 = 4 \times 44/4 = 1764$ (40 = 4 x 10)

$49^2 = 5(49 - 1)/1^2 = 5 \times 48/1 = 2401$ (50 = 5 x 10)

$49^2 = (1/2)\ (49 - 1)/01 = (1/2) \times 48/01 = 2401$ (50 = (1/2) x 100)

$209^2 = 2(209 + 9)/09^2 = 2 \times 218/81 = 43681$ (200 = 2 x 100)

$253^2 = 1/4)(253 + 3)/003 = (1/4) \times 256/009 = 64009$ (250 = (1/4)x 1000)

$507^2 = (1/2)(507+7)/007 = (1/2) \times 514/049 = 257049$ (500 = (1/2)x1000)

$311^2 = 3(311 + 11)/11^2 = 3 \times 322/\ 21 = 96721$ (300 = 3 x 100)
1

The usual carry over of excess digits, prefixing of zeroes on R.H.S. for deficit digits have to be done.

6.5 **CHECKING**

We can also check the squares of the numbers using the simple Vedic check method of checking the multiplication results. The following example (Refer Chapter 11, Pushp 1) review the simple procedure.

6.5.1 **ILLUSTRATION 17** Check the illustration 4.

$772^2 = 595984$

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Step 1: Convert the number into a single digit i. e.

$7 + 7 = 14,\ 1 + 4 = 5;\ 5 + 2 = 7.$

Step 2: Now square the single digit obtained in step 1 so $7^2 = 49$, and convert the square into a single digit so $4 + 9 = 13,\ 1 + 3 = 4$.

Step 3: Convert the result into single digit i.e.

$5 + 9 = 5,\ 5 + 5 = 10,\ 1 + 0 = 1;\ \ 1 + 9 = 1;\ 1 + 8 = 9;$

$9 + 4 = 13,\ 1 + 3 = 4.$

CHECK: The single digit obtained in step 2 must be equal to the single digit obtained in step 3. So

$4 = 4$ (O Kay)

PROBLEMS: Q.1. Find the square of the following numbers using Urdhva sutra.

i) 227^2 ii) 425^2 iii) 819^2 iv) 7336^2

v) 4787^2 vi) 67821^2 vii) 92416^2

Q.2 Find the square of the following numbers using Yavadunam sutra:

i) 9992^2 ii) 10006^2 iii) 100019^2

iv) 11102^2 v) 9989^2 vi) 99997^2

viii) 11115^2

Q.3 Find the square of the following numbers using Anurupena sub sutra.

i) 46^2 ii) 252^2 iii) 411^2 iv) 689^2

v) 428^2 vi) 559^2 vii) 4235^2

Q.4 Check the solutions of Q. 1, 2 and 3 using Vedic method.

Q.5 Convert the above numbers to Vinculum form and then square them using Urdhva sutra.

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CHAPTER-7

DIVISION

7.1 INTRODUCTION:

In Pushp-2 we have seen the use of three sutras namely Nikhilam (no.2), Paravartaya (No.4) and Urdhava (No.3) combined with Upsutra Dhwajank for division of numbers. Each one of these is most conveniently and efficiently aplicable in particular problems. Their applicability and working efficiency all the more increases when we freely use the Vedic Vinculum approach. We shall consider some additional working details in the present chapter.

7.2 NIKHILAM METHOD:

We use the Nikhilam sutra "Nikhilam Navatashcaramam Dashatah" (निखिलं नवतश्चरमं दशतः) to obtain the modified divisor (MD) as a first step. Further we use the MD for subsequent operations (Refer Chapter 9, Pushp-2). Further in illustration 8 and 9 some use of Vinculum approach has been illustrated and it has been shown that it is much more convenient to use the Vinculum numbers in place of bigger digits.

For example we have seen that we had to repeat the division process on the remainder portion in illustration 7 (Pushp -2) as the remainder obtained is too large. Let us do this problem again.

7.2.1 Illustration 1: Divide 34567 by 89.

Converting $34567 = 35\overline{433}$

```
                 /
        8 9 _/3 5 4 / 3 3
(MD)    1 1    3 3
                 8   8
                   7 7
               ----------
Repeating      3 8 7/1/2 4
operation                1 1
on remainder   ----------
               3 8 7/1/3 5
```

$387 + 1/35 = 388/35$

In Step 3 we obtained number 8 8. An important point is that it is possible to shift the excess digits on the R.H.S. places towards L.H.S. keeping in mind that the values on the extreme L.H.S. have to be contained within that column. Further we can rewrite any given number in almost infinite forms (Refer Chapter 12 on Multiple Choice). If we judiciously use this flexibility provided by the wonderful Vedic approach we can further simplify the working in Nikhilam method. That of course would demand greater mental alertness, poise, and practice. As a matter of fact Vedic methods give us the training to work with multi facet awareness which

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naturally leads to the all round development and efficiency of human brain and we function in the domain of all possibilities. Now here

$88 = 9\bar{2}$ and in 3rd step after obtaining 88, we can clearly see that we are getting a big digit in the third column (Here 7) which in turn shall lead to a bigger remainder. In 88 the 8 at the unit place can be written as $1\bar{2}$ and this excess 1 when shifted to the tenth place gives $8 + 1 = 9$. Therefore $88 = 8\ \overset{1}{\bar{2}} = 9\bar{2}$. In a way we can also look at this as shifting of excess digit 1 to the L.H.S. However it is not conveniently possible to put it as $1\ \bar{1}\ \bar{2}$, as this would require shifting of the left most digit.

```
                 /
        8 9 /3 5 4 / 3 3
(MD)    1 1    3 3
```

```
          9 / 2

            / 8 8
-----------
3 8 8 / 3 5
-----------
```

Therefore we can clearly see that the feeling which might have come while getting the initial introduction to Nikhilam method of division, that it is too clumsy and limited in its application is really incorrect.

7.2.2 **Illustration 2:** Divide 123212 by 88.

```
          8 8 / 1 2 3 2 / 1 2

(MD)      1 2   (1 2)
```

```
            2 8
              (4 8)

               5 2
                 0 / 0
                   / 0 0
-----------------
 1 4 0 0 / 1 2
-----------------
```

Therefore Q = 1400 and R = 12. If we do not convert the value in the 2nd step to Vinculum form, this shall become unnecessarily large and tedious.

7.3.2 **Illustration 3:** Divide 2341 by 89.

If we use the Vinculum approach and shift the digits freely,

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the working is drastically simplified (Refer illustration -11 Chapter 9 of Pushp-2).

```
          ___________
  8 9 /2 3 / 4 1
  1 1   (2   2)

          3   8
              6 6
        --------------
        2 6 / 2 7 0 0 0
        ---------------
```

Further dividing the remainder we can obtain the result in the decimal form.

```
(MD)  1 1      / 3 3 0 0 0
                 , 3 3
                     0 0
                       3 3
                ----------------------
              2 6 . 3 0 3 3 --------
                ----------------------
```

7.3 **PARAVARTAYA METHOD:**

Similarly we can do the shifting/carryover and freely use the vinculum forms to simplify the computation process even in the Paravartaya method.

7.3.1 **Illustration 4:** Divide 563478 by 112 upto 3 decimal places.

As a first step we put the divisor in the Vinculum form, get the modified divisor by using the Paravartaya sutra (Refer Chapter-10, Pushp 2) which says "Transpose and Apply".

```
563478   =  1 4 4 3 5 2 2
                  /
          1 1 2 / 1 4 4 3 5 / 2 2

            1 2     1 2

                  (5 10)
                   6  0
                      0 0

                     (3/6)

                      4 4

                        / 1 2
                ----------------------

                1 5 0 31 / 1 4
```


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Continuing

```
 1 2  /1 4 0 0 0
      1 2
        6 0
          4 8
            4 8
-----------------------
      1 5 4 4 4
1 5 0 3 1 .1 5 4 4 4
```

= 5031.05356

7.3.2 **Illustration 5 :** We know that when the divisor has not got 1 as the first digit we use the Anurupyena upsutra, divide or multiply the divisor by convenient number to satisfy this requirement for applying the Paravartaya Yojyet sutra. Finally the raw quotient is also modified using the Anurupyena upsutra. Here again by freely shifting the digits using Vinculum numbers lot of simplicity can be further obtained and fractional carryover to the remainder side can be avoided. (Refer Chapter 10, Pushp-2 & Illustration-9).

Divide 1233 by 224.

Dividing the divisor by 2 we get 112, therefore

```
1 1 2 / 1 2 / 3 3
 1 2     (1 / 2)
           2 / 8
             / 0 0
          -------------
       2) 1 0 /11 3
             5 / 113
```

However, if we had kept $1\bar{2}\bar{2}$ as it is we would have got 11 as the raw quotient and 1 as the remainder, but when we apply the Anurupyena upsutra we would get 5.5 and .5 of the diviso shall be shifted to the remainder portion.

.5*224 = 112 Therefore R = 112 + 1 = 113.

7.4 STRAIGHT DIVISION:

Here we use the Dhwajank upsutra which means "on the top of the flag". Further, when the flag digits are more than 1 we also use the Urdhva sutra (No. 3). The whole procedure is purely mental and after practice division can be done straight away in one line.

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7.4.1 **Illustration 6:** Divide 518 by 23.

```
23_/ 518
```

Step 1: We write it as follows

```
     F.D.
      3_/ 518
(operator)2
```

Here 3 is the flag digit.

Step 2: We put dots for the remainder portion, equal to the number of flag digits

```
 3_/ 51 : 8 :
2
```

Step 3: (i) Divide 5 by 2 we get 2 as first quotient digit and 1 as the remainder and place it before the next digit as

```
 3_/ 51 : 8 :
2    1
     --
     2
     --
```

So our next gross dividend is 11. Net dividend (ND) =11-3 x2=5.

Step 4: Divide net dividend (i.e. 5) by 2, we get 2 as the 2nd quotient digit and 1 as remainder and place it before 8 as,

```
 3_/ 51 : 8 :
2    11
     -----
       22
      -----
```

Next gross remainder is 18, as we have reached the remainder dots.

(ii) From 18 subtract

```
(3 x 2) =  -6
          ----
           12          So 12 is our final remainder.
          ----

 3_/ 51 : 8
2    11                Q  = 22
    --------
     22 : 12           R  = 12
    --------
```


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The whole process of step by step division as explained in the previous example can be done mentally and there is absolutely no need to write down the steps. By using this Vedic method of direct division we can get the total answer in one line, using the simple mental working. Let us try to understand the 8,9 & 10 illustrations by mentally doing the mental working as per the previous steps.

7.4.2 **Illustration 7:** In the previous example we have seen that out of the two digits of divisor we are actually dividing by using one digit and the flag digit is being used for simple readjustment of the gross dividend. Now let us take few examples of division by the three digit numbers.

Illustration 7: Divide 7142695 by 824.

Here the divisor is of 3 digits. In this case we place the last two digits of divisor as flag digits and adopt a slightly different modus operandi on Urdhva Tiryak lines as follows -

```
 24_/ 71426 : 95 :
8
```

The initial two steps are done as already explained, with last 2 digits marked off.

Step 1: (i) Divide 71 by 8 we get 8 and remainder is 7.

```
 24_/ 71426 : 95 :
8      7
      ---
       8
      ---
```

So 74 is the gross dividend.

(ii) From 74 we subtract the product of the 1st digit of flag number (i.e. 2) and the 1st quotient digit (i.e. 8). So we get 74 - (8 x 2) = 74 - 16 = 58. So 58 is our next net dividend (ND).

The subsequent steps have a slight modifications. We use the Urdhva sutra for adjustment of our gross dividend.

Step 2: (i) Divide 58 by 8 we get 6 and 10 as remainder.

So our next gross dividend is 102.

```
 24_/ 714 26 : 95 :
8      710
      ------
       86
      ------
```


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(ii) From 102 we subtract, (By Urdhva tiryak rule) gross products of two flag digits (24) and two quotient digits just obtained (86) i.e.

```
24
x       = 12 + 32
86      = 44
```

So our remainder is

```
102
-44
---
 58    (which is our next net dividend (ND).
----
```

Step 3: (i) On dividing 58 by 8 we get 6 and 10 as remainder.

Our next gross dividend is 106.

```
 24_/ 7 14 2  6 : 95 :
8     7 10 10
      --------
      8  6   6
      --------
```

(ii) From 106 we subtract, by Urdhva Tiryak rule gross products of two flag digit (24) and the last 2 quotient digits (66) i.e.

```
24
x      12  + 24  =  36
66
```

and the remainder is 106

```
-36
----
 70 which is our next net dividend.
-----
```

Step 4: (i) Dividing 70 by 8 we get 8 and 6 as remainder.

Our first gross remainder is 69, as we have reached the remainder portion.

```
  24_/ 7  1  4  2  6  : 95 :
8
        7 10  10  6
    ----------------------
      8  6  6  8  :
    ----------------------
```

(ii) From 69 we deduct by Urdhva Tiryak rule, gross product of two flag digits (24) and last 2 quotient digit 68 i.e.

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```
24
x    =  16 + 24  =  40
68
```

The next level remainder is 69

```
-40
----
 29
```

and 295 is therefore the gross R.

This has to be further corrected by the cross multiplication of 24 with 80. 2*0+8*4 = 32.

Net remainder R = 295-32 = 263.

This can also be obtained directly by obtaining the correction using the first stage of computations of Urdhva method.

```
24    =  432;  Therefore Net R  = 695 - 432 =  263.
68
---
 32
43
```

7.5 SPECIAL CASES:

In this process of division sometimes we find that the gross remainder left over is too small and the necessary adjustment product is bigger than it. This problem is easily handled by just using the next lower digit as our new quotient digit. This trial and adjustment situation is equally encountered in the division even in the present method. The following example illustrates this facet.

7.5.1 Illustration 8: Divide 601425 by 76.

```
  6_/ 60 1 4 2  : 5 :
7      11 6 3 5
      ----------
      7 9 1 3   : 3 7
      ----------
```

Here in the first division by 7 we can put down 8 as a quotient digit, but the remainder then left (4) will be too small for the subtraction expected at the next step (48). So we take 7 as the quotient digit and prefix the remainder 11 to the next dividend digit and then proceed as usual.

However if we are ready to use the Vinculum form of numbers, there is absolutely no need to go in for any trial digit. Let us do the same problem again.

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```
    6 : 6 0 1 4 2 : 5 :
7       4 0 3   5
  ---------------------
      8 1̄ 1 3 : 3 7 :
  ---------------------
```

Step 2: Raw divisor 41, net divisor 41 - 6*8 = -7 Therefore -7/7 = $\bar{1}$; R = 0.

Step 3: 04 - 6*$\bar{1}$ = 10

32 - 6*1 = 26

7.5.2 **Illustration 9:**

```
          5/ 2 4 8 2 9 6  8 : 5 :
6           6 5 6 10 9 5
          --------------------
            3 8 1 9 7 8 : 15
          --------------------
```

7.5.3 **Illustration 10:**

```
    24 / 3 2 3 4 : 3 7
3          5 8
         ---------------
           9 9 8 : 85:
         ---------------
```

OR

```
    24 /3 2 3 4 : 3 7
3        0 0 1̄ 0
         ---------------
           100 2̄  :  85
```

= 998 : 85

Remainder can be directly obtained in 1 step by following the first two steps of urdhava multiplication (R to L).

```
   24
  *02̄
  ----
  48̄   = -48
```

∴ R = 37 - (-48)

= 85.

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7.5.4 Illustration 11:

```
          ___________
  403 /4 5 3 : 4 4 5
3       1 2  2
       -------------
        1 3 3 :
```

Step 2 D = 15 - 4*1 = 11

It is equivalent to the cross products of the three digit. Dhwajank number (403) and the three digits of the quotient just obtained. But since we have computed only one digit so far (1) therefore effectively the three digits are 0, 0, 1 i.e. to say

```
4 0 3
0 0 1
----------------------
4*1 +0*3 +0*0  =  4
----------------------
```

Step 3: D = 23-(4*3+0*1) = 11 Cross multiplication

Similarly here we have the cross product of 403 with 0 1 3.

```
    4 0 3
    0 1 3
 -------------------------
= 4x3 + 0x3 + 1x0  =  12
 -------------------------
```

and so on. Therefore it is important to note that we always use the quotient digits (first obtained) equal to the number of digits in the flag number.

Step 4: Raw Remainder = 2445

For net remainder we can cross multiply 403 and 133 using only the 1st stage of computation of Urdhva method (Refer Pushp 1, Chapter 10).

```
 4 0 3    . . . . . .
*1 3 3    . . .;. .;.
--------
 1 5 9 9
```

R = 2445 - 1599

R = 846

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4.5.5 **Illustration 12:**

```
          212 /3 1 0 4 : 3 4 5 :
        4       3 0 1   1 9
              ----------------

                7 4 3̄ : 1 0 1 :
```

Step 2: D = 30 - [(2*7) + (0*1) + (2*0)]

= 16

Step 3: D = 04 -[2*4+1*7+0*2] = $\overline{11}$

Therefore Q 3 = $\bar{3}$ and R = 1 etc.

The remainder can be obtained in two alternative ways already explained. Finally Q = 737 and R = 101.

7.5.6 **Illustration 13:** If the number of digits in the divisor are large and we are interesteed to obtain only a limited number of digits of the quotient (For example when we are computing percentage of population etc.). It is interesting to conclude that many of the digits of the divisor as well as the dividend may not be touched even once and still we can get the exact required value by using this Vedic general method of division based on the upsutra Dhwajank and sutra Urdhva Tiryakbhyam (No. 3) .

What is the percentage value of 432461 of 32738678.

The desired percentage

= (432461 / 32738678)*100

```
          2738678 / 4:3 2 4 6 1 0 0 :
        3           1 2 3 0
                  ----------------
                  1 . 3 2 2
                  ----------------
```

= 1.32%

Step 2: D = 13 - 2*1 = 11

Step 3: D = 22 - [3*2 + 7*1] = 9

Step 4: D = 34 -[1*3 + 7*3 + 2*2] = 6

Notes: 1. Step 4 may be done simply to confirm the order of the next digit which if big may be suitably rounded off.

2. In step 3 we may be tempted to take 3 as the quotient and then in step 4 we shall have

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D = 04 -[1*3 + 3*7 + 3*2]

= - 26. Therefore Q4 = $\overline{8}$

and % = 1.3 3 $\overline{8}$ = 1.32 Ans.

3. However in case we are interested to compute further digits the simple Dhwajank procedure is continued.

Step 5: D = 06 -[2*2 + 7*2 + 3*3 + 1*8]

= - 29

Therefore Q 5 = -9 and R = $\overline{2}$ etc.

Similarly we can continue further.

COMMENTS:

This is another wonderful and unique feature of Vedic division method as we are required to do only the bare minimum essential amount of computations.

This is perfectly in tuned with the Vedic tradition where a human being, the roof and crown of Gods creation, is required to follow the path of divine bliss and fullfilment. The whole Vedic approach is towards poised and balanced minimization of efforts and maximization of results.

PROBLEMS:

1. Divide the following

   i) 108921 / 989 ii) 112372 / 332 iii) 123123 / 198

2. i) 232893 / 1138 ii) 123432 / 507 iii) 123753 / 198

3. Obtain result in decimal form.

   i) 21231 / 899 ii) 12353 / 87 iii) 21341 / 118

4. i) 3104345 / 4212 ii) 425372 / 693 iii) 323437 / 324

5. Obtain in decimal form upto 4 significant digits.

   i) 43 / 4123487 ii) 12543 / 39218

   iii) 456378956 / 3567821

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CHAPTER 8

SQUARING - DWANDWA YOGA

8.1 INTRODUCTION

In the previous chapter (6) we have learnt only the squaring of some particular type of numbers which are near to the base or subbase, using Yavadunam sutra.We have also learnt the Urdhava method for squaring.There is another method for squaring using Dwandwa yoga upsutra (or the Duplex combination). This sutra can be used in two ways, one is for squaring and second is the cross multiplication.

8.2 To find the square of the numbers we find the duplexes of the digits in sequence.

8.2.1 The Procedure:

We explain the method for computing duplexes in different levels.

Level 1: If we have a one digit number, to find the duplex (D) of one digit number we simply square the digit. For example 3, $D = 3^2 = 9$.

Level 2: For two digit number we find the duplexes of the digits as double the product of two digits, for example,

$$35 , D = 2 (3 \times 5) = 2 (15) = 30$$

Level 3: For three digit number we find the duplex as the sum of the square of the middle digit and double the product of the first and third digits as

For $3\ 2\ 7$; $D = 2 \times 3 \times 7 + 2^2 = 42 + 4 = 46$

Level 4: For four digit number we find the duplex as the sum of double the product of the first and fourth digits, and, double the product of the second and third digits as

For $1\ 4\ 5\ 6$; $D = 2 \times 1 \times 6 + 2 \times 4 \times 5$

$= 12 + 40 = 52$

Level 5: We extend the same procedure for finding the duplexes of five digit number

For $1\ 2\ 3\ 4\ 6$; $D = 2 \times 1 \times 6 + 2 \times 2 \times 4 + 3^2$

$= 12 + 16 + 9 = 37$

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Similarly we can extend it even for the larger digit numbers.

8.2.2 Squaring

The following illustration shall explain the steps in this method.

8.2.2.1 Illustration 1: Calculate 53^2.

Step 1: First we take the duplex of 1st digit 3, as described in level 1.

So $3^2 = 9$.

i.e. the square of the first digit.

$\therefore\ 53^2 = \ /\ \ /9$

Step 2: Now we take the duplex of first and second digits as described in level 2, so double the product of the two digits.

$\therefore\ 53 = 2 \times 5 \times 3 = 30$

Therefore $53^2 = \ /30\ /\ 9$

Step 3: Further we take the duplex of 2nd digit only, as in level 1. So square of 2nd digit is

$5^2 = 25$

$53^2 = 25/\ \ 0\ /\ 9$
$\quad\quad\ \ 3$

Step 4: Add the carry digits.

$53^2 = 25/\ \ 0\ /9 \quad = 2809$
$\quad\quad\ \ 3$

Hence $53^2 = 2809$

8.2.2.2 Illustration 2: Calculate 931^2.

Step 1: Duplex of 1st digit is

$1^2 = 1$

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$\therefore \quad \overset{\cdot}{931}^{2} = \quad / \quad / \quad / \quad /1$

Step 2: Duplex of 1st and 2nd digits i.e., double the product of two digits

We get for 31; 2 x 3 x 1 = 6

$\therefore \quad \overset{\cdot}{931}^{2} = \quad / \quad / \; 6 \; / \; 1$

Step 3: Duplex of 1st, 2nd and 3rd digits, level 3, we have

For 931; $D = 2 \times 9 \times 1 + 3^{2} = 18 + 9 = 27$

$\therefore \quad \overset{\cdot}{931}^{2} = / \quad / \; 27 \; / \; 6 \; /1$

Now we start the process of elimination.

Step 4: Duplex of 2nd and 3rd digits; double the product of two digits (level 2).

We have for 93; $D = 2 \times 9 \times 3 = 54$

$\therefore \quad \overset{\cdot}{931}^{2} = \; / \; 54 \; /27 \; / \; 6 \; / \; 1$

Step 5: Duplex of 3rd digit only (level 1)

We get $9^{2} = 81$

$\therefore \quad \overset{\cdot}{931} = 81 \; /54 \; / \; 27 \; / \; 6 \; / \; 1$

Carry over the excess digits = 866761

Hence $931^{2} = 866761$

8.2.2.3 Illustration 3: Calculate 8342^{2}.

Step 1: Duplex of first digit is $2^{2} = 4$

$\therefore \quad \overset{\cdot}{(8\ 3\ 4\ 2)}^{2} = / \; / \quad / \quad / \quad / \quad /4$

Step 2: Duplex of 1st and 2nd digit (level 2)

So for 42; D = 2 x 4 x 2 = 16

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$$\therefore\ (8342)^2 = \quad / \quad / \quad / \quad / \ {}^{1}_{6} \ / \ 4$$

Step 3: Duplex of 1st, 2nd and 3rd digits (level 3).

For 342; $D = 2 \times 3 \times 2 + 4^2 = 12 + 16 = 28$

$$\therefore\ (8342)^2 = \quad / \quad / \quad / \quad / \ {}^{2}_{8} \ / \ {}^{1}_{6} \ / \ 4$$

Step 4: Duplex of the first four digits. (level 4)

For 8342; $D = 2 \times 8 \times 2 + 2 \times 3 \times 4 = 32 + 24 = 56$

$$\therefore\ (8342)^2 = \quad / \quad / \quad / \ {}^{5}_{6} \ / \ {}^{2}_{8} \ / \ {}^{1}_{6} \ / \ 4$$

Now we start the limination process.

Step 5: Duplex of 2nd , 3rd and 4th digits (level 3) leaving 1st digit

For 834; $D = 2 \times 8 \times 4 + 3^2 = 64 + 9 = 73$

$$\therefore\ (8342)^2 = \quad / \quad / \ {}^{7}_{3} / \ {}^{5}_{6} \ / \ {}^{2}_{8} \ / \ {}^{1}_{6} \ / \ 4$$

Step 6: Duplex of 3rd and 4th digits i.e. 83; $2 \times 8 \times 3 = 48$

$$\therefore\ (8342)^2 = \ {}^{4}_{8} / \ {}^{7}_{3} / \ {}^{5}_{6} / \ {}^{2}_{8} / \ {}^{1}_{6} / \ 4$$

Step 7: Duplex of 4th digit only i.e. $8^2 = 64$

Step 8: Adding the carry digits we get square of the number

$$(8342)^2 = 64 / \ {}^{4}_{8} \ / \ {}^{7}_{3} / \ {}^{5}_{6} / \ {}^{2}_{8} / \ {}^{1}_{6} / \ 4$$

$$= 69588964$$

Hence $(8342)^2 = 69588964.$

8.2.2.4 Illustration 4: Calculate 256745^2 increasing the digits in

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each step

Step 1: 5 ; D $= 5^2 = 25$

Step 2: 45; D = 2 x 4 x 5 = 40

Step 3: 745; D = 2 x 7 x 5 + 4^2 = 70 + 16 =86

Step 4: 6745; D = 2 x 6 x 5 + 2 x 7 x 4 = 60 + 56 = 116

Step 5: 56745; D = 2 x 5 x 5 + 2 x 6 x 4 + 7^2 = 50 + 48 + 49

= 1 4 7

Step 6: 256745; D = 2 x 2 x 5 + 2 x 5 x 4 + 2 x 6 x 7

= 20 + 40 + 84 = 144

Eliminating the digits in each step.

Step 7: 25674; D = 2 x 2 x 4 + 2 x 5 x 7 + 6^2

= 16 + 70+ 36 = 122

Step 8: 2567; D = 2 x 2 x 7 + 2 x 5 x 6 = 28 + 60 = 88

Step 9: 256; D= 2 x 2 x 6 + 5^2 = 24 + 25 = 49

Step 10: 25 ; D = 2 x 2 x 5 = 20

Step 11: 2 ;D = 2^2 = 4

Step 12: Now adding the carry digits we get

256745^2 = 4/ 0/ 9/ 8/ 2/ 4/ 7/ 6/ 6/ 0/ 5
2 4 8 12 14 14 11 8 4 2

= 65917995025

Hence $(256745)^2$ = 65917995025

NOTE: There is absolutely no need to write these steps, and after a little practice the whole process is carried out mentally.

8.2.2.5 Illustration 5: Calculate $(4567423)^2$. All this process, we can also do mentally in one line as

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$$(4567423)^2 = 4^2 / 2x4x5 / 2x4x6 / 2x4x7 / / 5^2 / + 2x5x6 / /$$

$$+ \{ 2x4x4 / 2x4x2 / 2x4x3 / 2x5x3 / 2x6x3 / 2x7x3 / 2x4x3 / 2x2x3 / 3^2 \}$$
$$\{ 2x5x7 / + 2x5x4 / + 2x5x2 / + 2x6x2 / + 2x7x2 / + 2x4x2 / + 2^2 \}$$
$$\{ + 6^2 / + 2x6x7 / + 2x6x4 / + 2x7x4 / + 4^2 / / + 7^2 / \}$$

$$= \{ 16 / 40 / 48 + 25 / 56 / 32 / 16 / 24 / 30 / 36 / 42 / 24 / 12 / 9 \}$$
$$\{ +60 / +70 / +40 / +20 / +24 / +28 / +16 / +4 / \}$$
$$\{ / +36 / +84 / +48 / +56 / +16 / \}$$
$$\{ / +49 / \}$$

These two steps are optional

$$= 16/\ {}_{4}^{0}/\ {}_{7}^{3}/\ {}_{11}^{6}/\ {}_{13}^{8}/\ {}_{14}^{0}/\ {}_{14}^{1}/\ {}_{11}^{0}/\ {}_{8}^{0}/\ {}_{5}^{8}/\ {}_{2}^{8}/\ {}_{1}^{2}/\ 9$$

$$= 20861352860929$$

Hence $(4567423)^2 = 20861352860929$

8.2.2.6 Illustration 6: Calculate $(342564)^2$.

We can also square the number from L.H.S. to R.H.S. in the same manner.So

L.H.S. to R.H.S.

$$(342564)^2 = 3^2 / 2x3x4 / 2x3x2 / 2x3x5 / 2x3x6$$
$$/ +4^2 / +2x4x2 / +2x4x5 / +2^2 \}$$

$$\{ /2x3x4 / 2x4x4 / 2x2x4 / 2x5x4 + 6^2 / 2x6x4 / 4^2 \}$$
$$\{ /+2x4x6 / +2x2x6 / +2x5x6 / \}$$
$$\{ /+2x2x5 / + 5^2 / \}$$

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= 9/24 /12+16/ 30/ 36/ 24 / 32 / 16 / 40 / 48 /16

/ +16/+40/+48/ +24/ +60/ +36/

+ 4/+20/ +25/

= 9 / 4 / 8/ 6/ 0/ 2 / 1 / 6 /6 / 8 / 6
2 2 4 8 9 8 7 7 4 1

= 117350094096

DISCUSSIONS

The Vedic methods emphasis the mental working.The detailed steps outlined in the introductory illustrations need not be written down after a little practice.Further the speed of computations increases with practise.

The operation of bigger digits can be eliminated by converting to Vinculum form.

8.3 **CHECKING**

If we want to check the results then we may apply the sutra used for checking the multiplication results.(No.15)

8.3.1 Illustration: Check the illustration no. 3.

$(8342)^2 = 69588964$

Step 1: Convert the number 8342 into single digit i.e.

8 + 3 = 11, 1 + 1 = 2; 2 + 4 = 6; 6 + 2 = 8

Step 2: Square the single digit number obtained in step 1, i.e.,

$8^2 = 64$, and convert into the single digit.So

6 + 4 =10, 1 + 0 = 1.

Step 3: Now convert the result into single digit. So

6 + 9 = 15, 1 + 5= 6; 6 + 5 = 11, 1 + 1 = 2; 2 + 8 = 10,

1 + 0 = 1; 1 + 8 = 9; 9 + 9 = 9; 9+ 6= 6; 6 +4 = 10, 1 + 0 = 1

CHECK: The single digit obtained in step 2 and step 3 are equal. So the result is free from random error.

Hence 1 = 1 (O Kay).

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8.3.2 Illustration 8: Check the illustration No. 5.

$(4567423)^2 = 20861352860929$

Step 1: 4 + 5 = 9; 9+ 6 = 6; 6 + 7 = 13, 1 + 3 = 4; 4 + 4 = 8;

8 + 2 =10: 1 + 0 = 1; 1 +3 = 4.

Step 2: Square the above digit, i.e. $4^2 = 16$; 1 + 6 = 7

Step 3: The result

2 + 8 = 10, 1 + 0= 1; 1 + 6 = 7; 7 + 1 = 8; 8 + 3 = 11,

1 + 1 = 2; 2 + 5 = 7; 7 + 2 = 9; 9 + 8 = 8; 8 + 6 = 14,

1 + 4 = 5; 5 + 2 = 7.

Drop 9.

CHECK: 7 = 7 (O Kay).

Step 2 = step 3 The result is correct.

8.4 **VINCULUM FORM:** If we have bigger digits, converting to vinculum form shall reduce the computational effort.

8.4.1 Illustration 9: $2199^2 = 220\bar{1}^2$

$= 4 / 8 / 4 / \bar{4} / \bar{4} / 0 / 1 = 4835601$

8.4.2 Illustration 10: $3298^2 = 330\bar{2}$

$= 9 / \underset{1}{8} / 9 / \underset{1}{\bar{2}} / \underset{1}{\bar{2}} / 0 / 4$

$= 10883\bar{2}04 = 10876804$

8.4.3 Illustration 11: $20289^2 = 203\bar{1}\bar{1}^2$

$= 4 / \underset{1}{0} / 2 / \bar{4} / 5 / \bar{6} / \bar{5} / 2 / 1 = 4\ 1\ 2\ \bar{4}\ 5\ \bar{6}\ \bar{5}\ 2\ 1$

$= 411643521$

8.5 **ONE LINE POSITIVE RESULT:**

We have just seen how convenient it becomes to use the vinculum

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form of numbers. But it has the apparent deficiency of not giving the answer in one line. As same of the digits of the result are obtained in vinculum form. However, if we are interested in getting the positive result in one line we have just to convert the bar digit into its complementary digits and carry additional

$\bar{1}$ to the higher place as already explain in Pushp-2 (2nd edition).

8.5.1 Illustration 12: Square 2199.

$$2199^2 = 220\bar{1}^2 = 4/8/3_{\bar{1}}/5_{\bar{1}}/6/0/1 = 4835601$$

In step 3 we obtained the duplex as $2*2*\bar{1} + 0*0 = \bar{4}$ which is written as $\bar{1}6$ and $\bar{1}$ is shifted to the higher level and added to the next duplex number. In step 4 the duplex obtained is $\bar{4}$ this added to carryover, $\bar{1}$ gives us $\bar{5}$ which is used in the alternative form i.e. $\bar{1}5$. etc.

8.5.2 Illustration 13: Square 20289.

$$20289^2 = 203\bar{1}\bar{1}^2 = 4/1_{1}/1_{\bar{1}}/6/4_{\bar{1}}/3_{\bar{1}}/5/2/1$$

$$= 411643521$$

In step 3 we obtained duplex $\bar{5}$ which is used in the alternative form i.e. $\bar{1}5$, and we carry $\bar{1}$ to the higher level. In step 4 the duplex is $\bar{6}$ which when added to the carry $\bar{1}$ becomes $\bar{7}$. Using the alternative form as $\bar{1}3$, we get $\bar{1}$ carried to the higher level which when added to the 5th level duplex number of 5 gives $5 + \bar{1} = 4$. Similarly the 6th level duplex is obtained as $\bar{4}$ which is again used in its alternative form as $\bar{1}6$.

DISCUSSIONS:

1. Proceeding on the same lines, it is also possible to compute the results, even from left to right, while using the vinculum form of numbers, in one line. Here we shall shift the unit digit to the lower level as explained in Pushp-2 and ch. 6 of present Pushp.

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2. With this we come to the end of another very interesting chapter of Vedic Methods for the squaring of any number and checking of the solution obtained.

3. Once again we observe the emphasis on mental working. This approach when regularly practised leads to a more agile and alert mind, which in turn helps in total development of the cosmic computer.

PROBLEMS:

Q.1 Find the squares of the following numbers.

i) 43^2 ii) 236^2 iii) 3697^2 iv) 567^2 v) 2345^2

vi) 14679^2 viii) 268578^2 viii) 370415392^2

ix) 432656^2

Q.2 Find the squares of the following numbers using Vinculum approach.

i) 987^2 ii) 2689^2 iii) 68902^2 iv) 56326985^2

v) 3296586^2

Q.3 Square the following numbers.

i) 321.21^2 ii) 127.38^2 iii) 24.279^2

Q.4 Square the following numbers.

i) 0.231^2 ii) 0.0376^2 iii) $.00245^2$

Q.5 Check the results of Q.1 to Q.4.

Hints: (i) Q. 3 & 4, use the normal approach for fixing the decimal point.

(ii) Approximately round off the number and then fix the decimal point. Say for $.31^2 = .30^2$ (approx.) since $30^2 = 900$:

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CHAPTER 9

CUBING

9.1 INTRODUCTION

In the last chapters we have learnt how to square the number easily in one line. Similarly we can do the cubing of the numbers in one line using the sub sutra Anurupyena (आनुरूप्येण) which means proportionately. The following sections illustrate the step by step procedure of computing the cube of numbers.

9.2 SUCCESSIVE TERMS

In this method we find the successive digits of the resulting cube number. The method of computing these digits is simple. Let us take some examples.

9.2.1 Illustration 1: We find the successive terms of the number 11.

Step 1: The ratio of the two digits of the number is 1:1.

Step 2: First we find the cube of first digit as $1^3 = 1 \times 1 \times 1 = 1$, which is first successive term.

Step 3: For second term we multiply the second digit of the ratio with the first successive term.So 1 x 1 = 1, which is 2nd successive term.

Step 4: Again multiply the 2nd successive term with the second digit of the ratio we get 3rd successive term so 1 x 1 = 1.

Step 5: Further, 4th successive term is obtained by multiply the 3rd successive term by the second digit we get 1 x 1 = 1.

NOTE: So the four terms are in the geometrical proportion, in the ratio of the original digits.

4.2.2 Illustation 2: Find the successive terms of the number 14.

Step 1: The ratio is 1:4.

Step 2: Cube of first digit is $1^3 = 1$ (1st successive term).

Step 3: Multiply the second digit of the ratio (1:4) by the first successive term we get 2nd term, so 1 x 4 =4.

Step 4: Now multiply the 2nd digit of the ratio (here 4) with the 2nd successive term we get 4 x 4 = 16 (3rd successive term).

Step 5: For 4th term we multiply the 2nd digit by the 3rd successive term so we have 16 x 4 = 64.

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9.2.3 Illustration 3: Find the four successive terms of the number 22.

Step 1: The ratio is 1:1.

Step 2: Cube of 2 is 8 (1st term).

Step 3: Multiply 1 x 8 = 8 (2nd term).

Step 4: 8 x 1 = 8 (3rd term).

Step 5: Again 8 x 1 = 8 (4th term).

9.2.4 Illustration 4: Terms of 39.

Ratio is 1:3.

The successive terms are 27, 51, 153, 1377.

9.3 **TYPE I:** If the ratio between the digits is 1:X, where X is any number. The procedure is outlined with the help of the following examples.

9.3.1 Illustration 5: To determine 11^3.

Step 1: First we see the ratio of the two digits of the number i.e. 1:1.

Step 2: Now we write down the four successive terms in a row, whose ratio is same as that of the original two digits i.e. 1 1 1 1.

Step 3: Further we write down the double of the two middle numbers in the same column below the digits separately as (i.e. double of 1, 1 is 2, 2) so

```
1 1 1 1
  2 2
```

Step 4: Now we add the two rows and get the result. Hence

```
1 1 1 1
  2 2
-------
1 3 3 1
-------
```

11^3 = 1 3 3 1

9.3.2 Illustration 6: To determine 13^3.

Step 1: The ratio of the two digits of the number is 1:3.

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Step 2: The four successive terms are 1, 3, 9, 27 and write in one row

1 3 9 27

Step 3: Double the two middle digits (i.e. 3, 9) is 6 and 18, and write below the same digits so

```
1 3 9 27
  6 18
```

Step 4: Now add the two rows we get

```
1 3  9  27
  6 18
----------
1 9 27  27

OR  1 9  7   7
      2  2

    3
13  =  2 1 9 7
```

9.3.3 Illustration 7: Solve 17^3.

We write the cube as

```
  3
17  = 1   7   49   343
        14   98
      ------------------
      1  21  147   343

=     1    1    7    3
        2    14   34

= 4 9 1 3
```

9.3.4 Illustration 8: Find 18^3.

The ratio is 1:8.

```
  3
18  =  1  8   64   512
         16  128
       ---------------
       1 24  192   512

OR       1  4   2   2
           2  19  51

     =  5 8 3 2
```


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9.3.5 Illustration 9: To determine 24^3.

The ratio is 1:2. Successive terms are 8, 16, 32, 64. So

$$24^3 = \begin{array}{cccc} 8 & 16 & 32 & 64 \\ & 32 & 64 & \end{array}$$

$$\begin{array}{cccc} 8 & 48 & 96 & 64 \end{array}$$

OR

$$\begin{array}{cccc} 8 & 8 & 6 & 4 \\ & 4 & 9 & 6 \end{array}$$

$$= 1\ 3\ 8\ 2\ 4$$

NOTES The 4 successive terms are in the same ratio as that of the two digits.

9.4 **TYPE II:** If the ratio between the digits of the numbers is X:1, where X is any number, the method of cubing is same.The method to obtain the four successive terms is a little different, and is explained in the following examples. However the 4 terms are still in the geometric proportion.

9.4.1 Illustration 10: To find 31^3.

(a) First we find the four successive terms.

Step 1: The ratio is 3:1.

Step 2: The step 2 is same.We find the cube of the first digit of the given number. So $3^3 = 27$ which is 1st successive term.

Step 3: For 2nd term we divide the first term by the first digit of the number so 27 / 3 = 9 (2nd term).

Step 4: Further we divide the 2nd term by the first digit of the number i.e. 9 / 3 = 3 (3rd term).

Step 5: For 4th term again we divide the 3rd term by the first digit of the number or cube the second digit of the number i.e. 3 / $3 = 1$ or $1^3 = 1$.

Hence the four terms are 27, 9, 3, 1.Rest is the same. So

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$$31^3 = 27 \quad 9 \quad 3 \quad 1$$
$$\quad\quad 18 \quad 6$$
$$\text{---}$$
$$27 \; 27 \quad 9 \quad 1$$

OR

$$27 \quad 7 \quad 9 \quad 1$$
$$\quad 2$$

$$31^3 = 2\ 9\ 7\ 9\ 1$$

9.4.2 Illustration 11: To cube the number 81^3.

$$81^3 = 512 \quad 64 \quad 8 \quad 1$$
$$\quad\quad 128 \quad 16$$
$$\text{---}$$
$$512 \; 192 \quad 24 \quad 1$$

OR

$$512 \quad 2 \quad 4 \quad 1$$
$$\quad\quad 19 \quad 2$$

$$= 531441$$

9.4.3 Illustration 12: Find 111^3.

Hence

$$111^3 = 1331 \quad 121 \quad 11 \quad 1$$
$$\quad\quad\quad 242 \quad 22$$
$$\text{---}$$
$$1331 \quad 363 \quad 33 \quad 1$$

OR

$$1331 \quad 3 \quad 3 \quad 1$$
$$\quad\quad 36 \quad 3$$

$$= 1\ 3\ 6\ 7\ 6\ 3\ 1$$

9.5 **TYPE III:** If the ratio between the digits of the number is X:Y, where X and Y are numbers.This is the general case and can be understood by looking at the algebraic explanation.

$(a + b)^3 = a^3 + 3a^2b + 3b^2a + b^3$ which can be written as

$$a^3 + a^2b + ab^2 + b^3 \quad \text{-----} \quad (1)$$

$$+2a^2b + 2ab^2 \quad \text{-----} \quad (2)$$
$$\text{---}$$
$$a^3 + 2a^2b + 3ab^2 + b^3 \quad \text{-----} \quad (3)$$

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Both of the above methods are based on this simple split up of the well known expression for cube. Further we can clearly see that the successive four terms of equation (1) are in the same geometric proportion of the ratio between the digits of the given number.

In type I the ratio is 1:X and in type II, the ratio is X:1. Starting with cube of 1st part of the given number each of the successive term of the four parts is obtained by multiplying the previous term by the ratio X, for type I. However for type II each of the successive term is obtained by dividing the previous term by X.

In the general case (type III) the first term is cube of the first part (say a^3) and each of the subsequent 3 terms is obtained by multiplying the first term by the second part (b) and dividing by the first part. The following illustration shall elaborate the process.

9.5.1 Illustration 13: 23^3.

23^3 = 8 12 18 27
24 36
---
8 36 54 27

= 8 6 4 7 = 12167
3 5 2

9.5.2 Illustration 14: 34^3.

34^3 = 27 36 48 64
72 96
---
27 108 144 64

= 27 8 4 4 = 39304
10 14 6

Cube of a three digit number can be obtained in two steps.

9.5.3 Illustration 15: 121^3.

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$$121^3 = 1 \,/\, 21^3$$

$$= 1 \quad 21 \quad 441 \quad 9261$$
$$\quad\;\; 42 \quad 882$$
$$\quad 1 \quad 63 \quad 1323 \quad 9261$$

$$= 1 \quad 63 \quad 23 \quad 61$$
$$\quad\;\; 13 \quad 92$$

$$= 1 \;\; 7 \;\; 6 \;\; 1\;5\;6\;1$$ (two digits in each term are retained)
$$\quad\;\; 1$$
$$\quad\;\; -$$

$$= 1771561$$

where

$$21^3 = 8 \quad 4 \quad 2 \quad 1$$
$$\quad\;\; 8 \quad 4$$
$$\quad 8 \quad 12 \quad 6 \quad 1$$

$$= 8 \;\; 2\;6\;1$$
$$\quad\;\; 1$$

$$= 9\;2\;6\;1$$

Alternatively $12 \,/\, 1^3$

$$1728 \quad 144 \quad 12 \quad 1$$
$$\quad\;\; 288 \quad 24$$
$$1728 \quad 432 \quad 36 \quad 1$$

$$= 1728 \quad 2 \;\; 6 \;\; 1$$ (only one digit in each term is retained)
$$\quad\;\; 43 \;\; 3$$

$$= 1\;7\;7\;1\;5\;6\;1$$

9.6 **TYPE IV:** When numbers are closer to (greater or lesser than) the base.

9.6.1 Illustration 16: To find the cube of 107.

Step 1: Base is 100. Given number is greater than the base. Deviation from the base is 7.

Step 2: (i) Now double the excess digit i.e. 7 x 2 =14.

(ii) Add the number just obtained to the given number i.e., 107 + 14 = 121.This gives left most portion of the cube 121.

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Step 3: Further, we multiply the deviation of this number from the base (obtained in step 2 (ii))with the excess digit 7. (Please note that this deviation is nothing but 3 times the deviation of given number). We get the middle part of the cube i.e. 21 x 7 = 147.

Step 4: Now we cube the excess digits of the given number, which is the right hand portion of the answer.So cube of 7 = 343. Hence 121/147/343. Base is 100, so carry the excess digits as usual, and add i.e. 121/147/343.

OR 121/ 47 +3/ 43
+ 1

OR 122/50/43 = 1 2 2 5 0 4 3

9.6.2 Illustration 17: Evaluate 1015^3.

Step 1: Base is 1000. The deviation from the base is 015.

Step 2: (i) Double the deviation of the number i.e. 015 x 2 = 030.

(ii) Add the number obtained to the given number i.e. 1015 + 030 = 1045 (L.H.S. portion of the answer).

Step 3: Now multiply the deviation of this number from the base (obtained in step 2(ii))with the deviation of the given number i.e.045 x 015 = 675 (middle portion of the answer).

Step 4: Further, cube the excess portion of the given number i.e.

015^3 = 3375 (R.H.S. portion of the answer)

1	5	25	125
	10	50	

15^3 = 3375

Hence 1015^3 = 1045/675/3375

carry the excess digits as base is 1000.

= 1045/675 + 3/375 = 1045/678/375 = 1045678375

9.6.3 Illustration 18: To evaluate 10009^3.

Base is 10000. We write the answer in one line.

10009^3 = 10027/0243/0729
= 1002702430729

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9.6.4 Illustration 19: To evaluate 997^3.

Here the number is lesser than the base. Instead of doing addition we will do subtraction. The step by step procedure is given below.

Step 1: Base is 1000. Deficiency from the base is 003.

Step 2: (i) Double the deficiency of the number i.e. 003 x 2 = 006.

(ii) Subtract the number obtained (step 2(i)) from the given number i.e. 997-006 = 991 (R.H.S. portion of the answer).

Step 3: Now multiply the deficiency of the number obtained in step 2(ii) from the base with the deficiency of the given number, i.e. (1000 - 991) x 003 = 027 (middle portion of the answer).

Step 4: Further cube the deficiency digit of the given number i.e. $3^3 = 27$ with negative sign. So $\overline{027}$ is the R.H.S. portion of the answer.

Hence, $997^3 = 991/027/\overline{027}$

$= 991/027/\overline{1}973$

$= 991/027 + \overline{1}/973$

$= 991026973$

9.7 **TYPE V: VINCULUM** If we have the number with big digits then we can convert the number into Vinculum form and then apply the cubing method. We can apply the cubing method in different alternative ways.The following example shall illustrate the simple procedure which further simplifies the computation complexities.

9.7.1 Illustration 20: Cube the number 89.

Step 1: Convert the given number (89) into Vinculum form using Nikhilam sutra i.e.

$89 = 1\ \overline{1}\ \overline{1}$ (Vinculum form)

Step 2: Break the Vinculum form of the number into two parts as we do in the cubing of any number i.e. $1/\overline{1}\ \overline{1}$.

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Step 3: Now we follow the cubing procedure directly.

$$89^3 = 1/\bar{1}\ \bar{1}^3 = 1\quad \bar{1}\ \bar{1}\quad 1\ 2\ 1\quad \bar{1}\ \bar{3}\ \bar{3}\ \bar{1}$$

$$\bar{2}\ \bar{2}\quad 2\ 4\ 2$$

$$1\quad \bar{3}\ \bar{3}\quad 3\ 6\ 3\quad \bar{1}\ \bar{3}\ \bar{3}\ \bar{1}$$

$$= 1\ \bar{3}\ 0\ 5\ 0\ \bar{3}\ \bar{1} \quad = \quad 7\ 0\ 4\ 9\ 6\ 9$$

We can see how convenient it is to handle the smaller digits.

Alternatively: We can break the vinculum form of number as $1\ \bar{1}/\bar{1}$ and then use the same procedure, we get

$$1\ \bar{1}\ /\ \bar{1}^3 = 1\ \bar{3}\ 3\ \bar{1}\quad \bar{1}\ 2\ \bar{1}\quad 1\ \bar{1}\quad \bar{1}$$

$$\bar{2}\ 4\ \bar{2}\quad 2\ \bar{2}$$

$$1\ \bar{3}\ 3\ \bar{1}\quad \bar{3}\ 6\ \bar{3}\quad 3\ \bar{3}\quad \bar{1}$$

$$= 1\ \bar{3}\ 0\ 5\ 0\ \bar{3}\ \bar{1}$$

$$= 704969$$

NOTES:

1. The method of cubing can also be applied from R.H.S. to L.H.S.

2. We can directly use the same method by using the base.

3. Direct method of ratio for cubing can also be used.

4. Once again, as usual we find the 3 alternative methods available in the Vedic Maths approach, giving a choice to the user.

9.7.2 Illustration 21: Cube the number 79.

$$79 = 1\ \bar{2}\ \bar{1}$$

$$79^3 = 1\ \bar{2}\ \bar{1}^3$$

$$= 1/\bar{2}\ \bar{1}^3 = 1\quad \bar{2}\ \bar{1}\quad 4\ 4\ 1\quad \bar{9}\ \bar{2}\ \bar{6}\ \bar{1}$$

$$\bar{4}\ \bar{2}\quad 8\ 8\ 2$$

$$1\quad \bar{6}\ \bar{3}\quad 13\quad 23\quad \bar{9}\ \bar{2}\ \bar{6}\ \bar{1}$$

$$= 1\ \bar{5}\ 0\ \bar{7}\ 1\ \bar{6}\ \bar{1}$$

$$= 493039$$

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NOTE: We can also apply the alternate method for cubing

$$1\ \bar{2} / \bar{1}^{3} = 1\ \bar{5}\ 2\ \bar{8} \quad \bar{1}\ 4\ \bar{4} \quad 1\ \bar{2} \quad \bar{1}$$

$$\bar{2}\ 8\ \bar{8} \quad 2\ \bar{4}$$

$$1\ \bar{5}\ 2\ \bar{8} \quad \bar{2}\ 1\ \bar{2} \quad 3\ \bar{6} \quad \bar{1}$$

$$= 1\ \bar{5}\ 0\ \bar{7}\ 1\ \bar{6}\ \bar{1} \quad = \quad 493039$$

9.8 CHECKING METHOD

To check the cube of any number we apply the same method as we use in the multiplication. Only a single step is slightly different. Instead of multiply the single digits obtained, we do the cube of the single digit. The following illustrations shall explain the method.

9.8.1 Illustation 22: Check the cube of 24^{3}.

$$24^{3} = 8 \quad 16 \quad 32 \quad 64$$

$$32 \quad 64$$

$$8 \quad 48 \quad 96 \quad 64$$

$$= 8\ 8\ 6\ 4 \quad = \quad 13824$$

$$4\ 9\ 6$$

CHECK

Step 1: First we convert the given number into a single digit. Here 2 + 4 = 6.

Step 2:(i) Now we find the cube of the single digit obtained in step 1. i.e. $6^{3} = 216$.

(ii) Convert the cube of the single digit (216) into the single digit as 2 + 1 = 3; 3 + 6 = 9.

Step 3: Now we convert the result into the single digit.So 13824 is 1 + 3 = 4; 4 + 8 = 12, 1 + 2 = 3; 3 + 2 = 5; 5 + 4 = 9.

Step 4: Compare the two single digits obtained in step 2(ii) and step 3. If they are equal then our result is correct. As 9 = 9 (O-kay).

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9.8.2 llustration 23: Check 96^3.

$$96^3 = 8\ 8\ 4\ 7\ 3\ 6$$

Step 1: Convert the number into a single digit i.e. 9 +6 = 15, 1 + 5 = 6.

Step 2:(i) Cube the single digit obtained i.e., $6^3 = 216$.

(ii) Convert into a single digit, so 2 + 1 + 6 = 9.

Step 3: Convert the result into a single digit i.e. 8 + 8 = 16, 1 + 6 = 7; 7 + 4 = 11, 1 +1 = 2; 2 + 7 = 9; ignore 9; 3 + 6 = 9.

Step 4: 9 = 9 Step 2(ii) = Step 3. (O Kay)

DISCUSSION:

The simple Vedic check method is applicable even for cubing of numbers, and requires just a few seconds for mentally checking the correctness of the result obtained.

PROBLEMS

Q.1 Cube the following numbers and identify the sutra used.

i) 19^3 ii) 26^3 iii) 61^3 iv) 131^3

v) 43^3 vi) 128^3 vii) 89^3 viii) 1004^3

ix) 98^3 x) 9989^3 xi) 100015^3

Q.2 Change the numbers of Q.1 into vinculum form and then cube.

Q.3 Check the result obtained, by using the Vedic check method.

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CHAPTER 10

SQUARE ROOTING

10.1 INTRODUCTION

To find the square root of the number using the vedic sutra is very simple, easy and interesting. The sub sutra named Dwandwa Yoga (द्वन्द्वयोग) or duplex process has two meanings. First one is by squaring and the second one is cross multiplication.This sutra is very easy to apply. Using this sutra we can also find the squares of the numbers. In Chapters 6 & 8 we have seen different methods of squaring.

10.2 THE PROCEDURE

The use of the above sutra is explained below :

Computing Duplexes:(Refer Chapter 8)

Level 1: If we have a single digit number than we take square of number.

Example 4 then duplex is $4^2 = 16$

Level 2: We have a two digit number then twice of the product of the two digits of the number.

For example for 23; duplex = 2 x 2 x 3 = 12 etc.

10.2.1 Square Root

Using the above method the step by step process of computing the square root is illustrated in the following examples:

10.2.2 Illustration 1: Find the square root of 4489.

Step 1: First we make the pairs of two digits each from the given number R.H.S to L.H.S. and write as

44 : 89 :

The first pair of two digits from L.H.S. is written outside the dots and rest of the pairs are written in between the dots.

Step 2: Now we find the square of the first nine natural numbers as 1, 4, 9, 16, 25, 36, 49, 64, 81 and choose one of them which is most suitable and lesser than the first pair.

(i) In this case square of 6 is suitable.So 6 is the first quotient digit. After this step we find the remaining part of the square root using straight division process. So subtract the square of 6 (i.e. 36) from 44 we get (44 - 36 = 8) 8 as a remainder which is prefixed as the first digit of the next pair (i.e. 8). Hence 88 is the dividend.

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(ii) For finding the divisor we double the first quotient digit (i.e. 2 x 6 = 12) obtained in step 2 (i) and write as follows

```
      44 :  89 :
         8
   12
      -----------
        : 6 :
      -----------
```

So 12 is our divisor. The entire division is done by 12.

Step 3: Now divide 88 by 12 we get 7 as a quotient and 4 as the remainder which is prefixed to the next digit 9. So 49 is our next gross dividend.

```
        44 :  8  9 :
   12:        8 4
      -------------
        : 6 : 7 :
      -------------
```

Now subtract the square of the quotient digit (i.e. $7^2 = 49$) obtained in (step 3 (i) for the gross dividend 49 we get (49 - 49 = 0) 0 as quotient and 0 as remainder. Hence the square root is 67, which is an exact square.

DISCUSSION:

(1) We have just seen how easy it is to find the square root of any big number.

(2) If a number contains n digits

(i) If n is odd than the square root of a number shall have $\frac{n+1}{2}$ number of digits.

(ii) If n is even than the square root of number shall have $\frac{n}{2}$ digits.

(iii) If the number is of pure decimal then the number of digits in the square, is double, that in the square root.

(iv) If the square root is not an exact number, the decimal shall be put after the number of digits equal to the number of pairs formed.

10.2.3 Illustration 2: Find the square root of 567009.

Step 1: First we make the pairs of the number and place in between the dots as shown.

```
      56 : 70 09 :
```


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Step 2: Now we find the suitable digit for first pair. Here 49 is most suitable lesser number. So 7 is the first quotient digit. Subtract 49 from 56 we get 7 as a remainder, which is prefixed to the next digit (7) so 77 is the dividend.

The divisor is double of the first quotient digit 7; we get 14. So 14 is the divisor and we write as

```
 56 : 70 09 :
    :7
14:
-------------
    : 7 :
-------------
```

Step 3: Divide 77 by 14 we get 5 as a quotient digit and 7 as a remainder, which is prefixed to the next digit 0.So 70 is the next gross dividend.

```
  56 : 70 09 :
14:  :77    :
-------------
  : 7 : 5   :
-------------
```

Subtract the square of the quotient digit 5 (i.e. $5^2 = 25$) (level 1) from the gross dividend (70) to get the net dividend. (70 - 25 = 45; 45 is the net dividend).

Step 4: Now divide 45 by 14 we get 3 as the next quotient digit and 3, the remainder which is placed before the next digit 0. So 30 is next gross dividend.

```
    56 : 7 0 0 0 :
14:    :7 7 3    :
---------------
    : 7 : 53 :
---------------
```

Subtract the double of the product of two quotient digits 5 & 3 (using Dwandwa yoga upsutra (level 2) from 30, we get the net dividend. 2 x 5 x 3 = 30 ;ND= 30 -30 = 0.So 0 is net dividend.

Step 5: Now 0 is not divisible by 14. So 0 is both the quotient and remainder which is placed before the next digit 9.

Now 09 is the gross dividend.

```
  56 : 7 0 0 9 :
14:  :7 7 2 0  :
---------------
   7 : 5 3 0 0 :
---------------
```


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Subtract the sum of the double of the product of the quotient digits 5 and 0 and the square of the middle digit 3 (i.e. $2 \times 5 \times 0 + 3^2 = 9$) from 9 (level 3); we get (9 - 9 = 0) 0 as quotient and 0 as remainder.

Hence the square root of 567009 is 753. This is an exact square root case.

10.2.4 Illustration 3: Find the square root of 17.

```
 17 : 0 0 0 0 0 0 0 0
8:  :1 2 3 2 2 6 7 3
---------------------
: 4 :1 2 3 1 0 5 6
---------------------
```

the net dividend at each stage is

10

$20 - 1^2 = 20 - 1 = 19$

$30 - 2 \times (1 \times 2) = 30 - 4 = 26$

$20 - (2 \times (3x1) + 2^2) = 20 - 10 = 10$

$20 - (2 \times (1x1) + 2 \times (2 \times 3) = 20 - 14 = 6$

and so on. Hence the square root of 17 is 4.1231056 which is not an exact square root.

10.2.5 Illustration 4: Find the square root of 229441.

Step 1: First we make the pairs of two digits each of the number from R.H.S. to L.H.S. and write as

22 : 94 41:

The first pair from L.H.S. is written before the first dot and other pairs are written in between the dots.

Step 2:(i) Now we find the square of the first nine natural numbers as 1, 4, 9, 16, 25, 36, 49, 64, 81. In this case we take 4 as the first digit of the square root because 16 is last smaller number than 22 and the remainder is 6 which is prefixed to the next number here 9).After this step we find the remaining part of the square root, using straight division process.

So 69 is our next dividend.

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(ii) For finding the divisor we double the first digit of the square root.

```
  22 : 9 4 4 1 :
8:   6
--------------
   : 4 :
--------------
```

Here double of 4 = 8.So 8 is our divisor and we write as

```
   22 : 94 4 1 :
8:    6
---------------
      : 4 :    :
---------------
```

Step 3: We divide 69 by 8 we use 7 instead of 8 (if we take 8 as the quotient digit remainder is 5, giving gross = 54. After using Dwandwa yoga sutra for getting net dividend, the square of the quotient digit (level 1) is greater than the gross dividend, 8 x 8 = 6 4)So 7 is the second quotient digit and 13 as the remainder which is prefixed before 4. So 134 is our next gross dividend.

```
8 :  22 : 9  4 4 1 :
        :6 13      :
--------------------
   : 4 : 7         :
--------------------
```

(ii) Subtract the square of the quotient digit ($7^2 = 49$) 49 from the gross dividend 134, we get (134 - 49 = 85) 85 as the net dividend.

Step 4: Now 85 is divided by 8 we get 9 instead of 10 as a third quotient digit and 13 as a remainder, prefixed before the next digit 4. So 134 is the next gross dividend.

```
  22 : 9  4  4 1 :
8:    6 13 13   :
------------------
 : 4 : 7  9      :
------------------
```

(ii) Now subtract the double of the product of the quotient digits (level 2) as (2 x 7 x 9 = 126) 126 from the gross dividend 134, we get (134 - 126 = 8) 8 as the next net dividend.

Step 5: Again we divide 8 by 8.We use 0 instead of 1 (see step 3). So 0 is the next quotient digit and 8 is the remainder.

```
  22: 9  4  4 1 :
8:   6 13 13 8
--------------
 : 4 : 7 9. 0 :
--------------
```


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81 is the next gross dividend.

(ii) Again we subtract the duplex of the three quotient digits (. .

$2 \times 7 \times 0 + 9^2 = 81$) (level 3) from 81 (gross dividend). We get (81 - 81 = 0). So ND = 0 and we get 0 as both the quotient and remainder. Hence our work has been completed.

So the square root of 229441 is 479 which is a perfect number.

10.2.6 Illustration 5: Find the square root of 16641.

```
  1 : 6 6 4 1 :
2:  : 0 2 4 8
---------------
    : 1 : 29.00:   (complete)
---------------
```

The result is 129.

NOTES:

(i) After making the two digit pairs of the number from R.H.S. to L.H.S. if a single digit is left then it is counted as a pair (of a single digit).

(ii) When we divide 6 by 2 after step 2, we write 2 as a quotient digit instead of 3, if we write 3 as Q we get remainder 0 and the next gross dividend is 06 which is smaller than the square of the Q digit 3 (i.e. 6-9) which would give the negative dividend value. So we take 2 as the quotient digit.

(iii) Due to the same reason we do not divide the net dividend 8 by 2 (obtained in step 5 (ii)).

VINCULUM APPROACH: We can also proceed by using Vinculum numbers.

```
 1 : 6 6 _ 4  1 :
     0  1  0
2 :
------------------

 1 : 3 1 0 0
------------------
```

Square root = $13\bar{1}$ = 129; is the exact square root.

Net dividend

$1^2 = 1$

$2 \times 1 = 2$ divisor

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$06 - 3 \times 3 = \bar{3}$

$\bar{1}\ 4 - 2 \times 3 \times \bar{1} = -6 + 6 = 0$

$01 - (2 \times 3 \times 0 + \bar{1} \times \bar{1}) = 1 - 1 = 0$

10.2.7 Illustration 6: Find the square root of 1145678.

```
  1 : 1   4  5  6  7  8
2:  :0  1  0  5  1  5
------------------------
  : 1 : 0 7 0 . 36  (incomplete)
------------------------
```

The net dividend at each stage is

0 1

14 - 0 = 14

5 - 0 = 5

56 - 49 = 7

17 - 0 = 0 and so on.

Hence the square root of 1145678 is not a perfect number.

So $\sqrt{1145678} = 1070.36$ --------

10.3 SQUARE ROOT OF DECIMAL NUMBERS

The Vedic method for finding the square root of decimal numbers is perfectly the same.

10.3.1 llustration 7: Find the square root of 547.56

```
    : 5 : 4 7 . 5 6
4 :     12   2 1
---------------
  : 2 : 3 . 400 :
---------------
```

The single digit 5 on L.H.S. is counted as a pair by itself.The net dividend at each stage is

1 4

$2\ 7 - 3^2 = 18$

$2\ 5 - 3 \times 4 \times 2 = 1$

$1\ 6 - 4^2 = 0$

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Hence the square of the number is 23.4.

NOTES: As there are 2 pairs on L.H.S. of the decimal point we have 2 digits on L.H.S. of decimal point. The remaining digits shall be after the decimal point (on R.H.S.).

10.3.2 Illustration 8: Find the square root of the number .000015.

First we make the pairs of the number as .00 00 15 gives 15 as the first non zero digit pair. The square root of the two pair of zero is .00 according to the discussion 2(iii).So we find only the square root of 15.

Hence

```
        : 1 5 : 0  0  0  0
6 :       6   12 14 10
----------------------
  : 3 : 8 7 3 0 - -
----------------------
```

Hence the square root is .0038730 of the given number.

10.3.3 Illustration 9: Find the square root of 1 2 4 3 . 5 9.

```
     : 1 2 : 4 3 . 5 9 0  0
           3 4 / 6 9 11 16
6:
-------------------------
 : 3 : 5.26 4 7 ------
-------------------------
```

Hence the square root is 35.2647 etc.

10.4 **VINCULUM APPROACH:**

10.4.1 Illustration 10: Let us try to solve the illustration 8 again by using Vinculum approach.

```
. 00 00 15  for 15
      15: 0 0 0 0 0

          6 6 3̄ 6 3̄
    6
---------------------

     3 . 9 3̄ 3 0 2      =   3.87302  etc.
---------------------
```

Hence .00387302 is the square root.

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10.4.2 Illustration 11: Solve again illustration 9 using Vinculum.

: 12 : 4 3 . 5 9 0 0 0

34 0 $\bar{1}$ 2 0 0

6

---

3 : 5 3 $\bar{4}$ 5 $\bar{1}$ $\bar{6}$ = 35.26484 etc.

10.5 **DISCUSSION:**

1. In this chapter we have studied the straight, simple and easy Vedic method of finding the square root of any number.

2. The Dwandwa Yoga upsutra for squaring of numbers has been employed again even for computing the square root.

3. The step by step Vedic Method of square rooting supersedes the present methods in many respects.

4. The checking method can be easily and directly applied for checking the correctness of results in case of the exact square roots.

PROBLEMS

Q.1 Find the squre root of the following numbers upto three decimal places:

i) 1552542907 ii) 753128 iii) 7410756 iv) 8009240270

v) 1882405 vi) 738915489 vii) 25745476 viii) 45319824

ix) 984519837 x) 92945241 xi) 1360996

Q.2 Find square root of the following numbers upto four decimal places.

i) 524.134 ii) 64676.23 iii) 2364.4502 iv) 567.3498

v) 89.24678

Q.3 Check the square root of the exact numbers.

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CHAPTER 11

PRACTICE AND PROPORTION

11.1 INTRODUCTION

In the previous Pushps 1 & 2, we have learnt the simple superfast Vedic methods of performing addition, subtraction and multiplication operations.We have also learnt the simple applications of the vedic check sutra 'Samucchya' for confirming the correctness of our computations.We have also learnt how 'Urdhva' sutra can be employed even for the compound multiplication.There is another class of problems where we have to work with different units. This type of problems are grouped under the practice and proportion.The 'Urdhva' sutra can be very efficiently employed for solving this class of problems as well.

11.1 THE FIRST CONVENTIONAL METHOD

At present we have two alternative methods of handling this type of problems.The first is by using the simple proportions where we convert both the quantities into a single-unit vulgar fraction and then multiply.The resulting complex fractions are then normalised in the usual long, time consuming and tedious process of divisions and multiplications.Let us solve one problem by this approach.

11.2.1 Illustration:

We consider the problem of carpeting the flooring of a room. If PVC mat costs Rs. 7.25 per foot, what is the cost of 5 yards 1 feet and 9 inch long mat ?

Comment: The whole process is rather too long and difficult. The details are too well known to be repeated here.

11.3 THE SECOND CONVENTIONAL METHOD

As an alternative to the above approach involving vulgar fractions we can use the method of aliquot parts.

1 feet long mat costs	= Rs. 7.25	
1 yard long mat costs	= Rs. 7.25 x 3	= 21.75
5 yards	= Rs. 108.75	
1 feet long mat costs	= Rs. 7.25	
6 inches long mat costs	$= (-)\ 3\frac{1}{2} - 6\frac{1}{2}$	
3 inches long mat costs	$= (-)\ 1\frac{1}{2} - 1\frac{1}{4}$	
Total cost of 5 yards 1 feet 9 inches		$= 121.43\frac{3}{4}$

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Comment: This alternative is also in no way simple and easy.

11.4 THE VEDIC METHOD

In the vedic method, we use the 'Urdhva' sutra.

$$(5 \times 3 + 1)A + 9 \qquad A = 12$$
$$7\ B + 25 \qquad B = 100$$

$$112AB + 63B + 400A + 225$$

$$= 112AB + 5 \times 12B + 3B + 4 \times 100A + \frac{75}{4} A \qquad (1)$$

$$= 112AB + 5AB + 4AB + \frac{3 \times 100}{12} A + \frac{3}{4} - A \qquad (2)$$

$$= 121AB + 25A + 18 - \frac{3^{\circ}}{4} A \qquad (3)$$

$$= 121AB + 43 - \frac{3}{4} A$$

= 121 Rs. 44 Paise.

We should note that since the rate is given per feet therefore it is essential to have A in every term. We bring A in the terms by dividing them by the value of A(A=12).

Comment: The whole process becomes a simple affair which is a joy rather than an infliction.

11.5 ILLUSTRATION 2

The cost of running a motor is Rs. 8.09 per hour. What would be the total cost of running for 5 hours 20minutes.

In this case, the unit sizes are 100 and 60 respectively.

Therefore $8A + 9$ $\qquad A = 100$
$5B + 20$ $\qquad B = 60$

$$40AB + 160A + 45B + 180$$

$$= 40AB + 2 \times 60A + 40A + 45B + 3 \times 60 = 42AB + 40 \times 100 + 48B$$

$$= 42AB + \frac{4000}{60} B + 48B = 42AB + 66\frac{2}{3} + 48)B$$

$$= 42AB + AB + 14\frac{2}{3} B$$

=Rs. 43 and 15 paise.

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Comment: Once again since the rate is given per hour it is important to bring in B in every term (by dividing the constant terms by 60). Further, the coefficients of AB are essentially the cost in rupees and those of B are the paise.

This is also much simpler then the conventional process and with little practice the number of steps can be reduced to just three.

PROBLEMS

Q.1 Find the area of a rectangular field whose length & breadth are 6 m 86 cm and 3 m 95 cm. If the cost of binding a wire around the whole field is Rs. 3.25 per m. then find the total cost.

Q.2 Find the area of the following rooms whose dimensions are given. If the P.V.C. Mat cost is Rs. 15.50 per sq.ft. then find the total cost.

	length	Breadth
i)	5′8"	3′7"
ii)	9′2"	5′9"
iii)	6′5"	6′5"
iv)	4 yd.2′3"	3 yd 4′5"

Q.3 If the cost of a cloth is Rs. 135.60 per m then find the cost of the cloth 16 m 85 cm long.

Q.4 Mean cycling speed is 3 km 5 m per hour. How much distance he shall travel in 8 hours 20 minutes.

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CHAPTER 12

VEDIC NUMERICAL CODE

12.1 INTRODUCTION

It may be a matter of interest to note that Vedic scholars of ancient time did not use figures (for big numbers) in their numerical notations but preferred to use Sanskrit alphabets to represent various numbers. This was perhaps done to facilitate the mathematical knowledge stored and transferred on to subsequent generations by way of language, understood to mankind.Thus presentation of their knowledge was better managed and understood by subsequent followers.Sutras and Sanskrit verses are in fact treasures of such kind which can be easily decoded and assimilated even to the modern world.

12.2 RECENT RESEARCH

It is interesting to observe that the latest research of the modern scientists is once again bringing before humanity the perfections of the sanskrit language and lot of systematic effort is being done, the world over, to unearth and decode the scientific knowledge contained in the ancient sanskrit texts with special reference to THE VEDAS. In this context, it is interesting to point out the honour being given to the Sanskrit language by the American Association for Artificial Intelligence. The artificial intelligence is the special research area of computers used for the robots - the mechanized servants. The full cover page of the AI magazine (volume 6 No. 1 Spring 85), an official publication of the American Association of Artificial Intelligence is decorated with Sanskrit shlokas. This particular issue also contains a research article of 10 pages by Dr. Rick Briggs, who is a computer scientist at NASA, American Space Research centre, CALIFORNIA, on,'Knowledge representation in Sanskrit and Artificial Intelligence'. Dr.Rick Briggs says, there is a wide spread belief that Natural languages are unsuitable for the transmission of any ideas that artificial languages can render with great precision and mathematical rigor. But this dichatomy which has served as a premises underlying much work in the area of linguistics and artificial intelligence, is a false one. There is at least one language Sanskrit which for the duration of almost 1000 years was a living spoken language with a considerable literature of its own. Besides works of litrary values, there was a long philosophical and grammatical tradition that has continued to exist with undiminished vigor until the present century. Among the accomplishments of the grammmarians can be reckoned a method for paraphrasing Sanskrit in a manner that is identical not only in essence but in form with current work in artificial intelligence.This article demonstrates that a natural language can serve as an artificial language also, and that <u>much work in AI</u> has been <u>reinventing a wheel millenia old</u>. It is interesting to speculate as to why the Indians found it worthwhile to pursue studies into unambigous coding of natural language into semantic elements. It is tempting to think of them as <u>computer</u>

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scientists. Let us not forget that among the great accomplishments of the Indian thinkers were the invention of zero and of the binary number system a thousand years before the west reinvented them". It will be useful to point out here that all the calculators and computers of the world are based on the binary number system. Further the computers are also using the Vedic Vinculum system in the IC circuits.

THE VEDIC CODE

The total code system is given in its very simple poetry as follows :

" कादिनव टादिनव पादि पच्चक यार्धष्टक क्षःशून्यम् "

The code system in Hindi could be written down as:

क	ट	प	य	1
ख	ठ	फ	र	2
ग	ड	ब	ल	3
घ	ढ	भ	व	4
ङ	ण	म	श	5
च	त	-	ष	6
छ	थ	-	स	7
ज	द	-	ह	8
झ	ध	-	-	9
	क्ष		-	0 ॥शून्यम्॥

Note: Further न and are also be coded as 5.

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This code system can also be written in English as follows:

(i)	ka, tta, pa and ya all denote	1,
(ii)	kha, dah, pha, ra all denote	2,
(iii)	ga, da, ba, la all denote	3,
(iv)	gha, dha, bha, va stand for	4,
(v)	gna, na, ma and sha denote	5,
(vi)	ca, ta and ksha denote	6,
(vii)	cha, tha, sa denote	7,
(viii)	ja, da and ha stand for	8,
(ix)	jha, dah denote	9 and
(x)	ksa (or Ksudra for shunyam) for	0

The vowels are ignored and in case of conjunct consonants, the last consonant is alone counted.

12.4 ILLUSTRATION:

Thus pa pa is 11, ma ma is 55, ta ta is 11, mary is 52 and so on.

It was left to the author to select proper consonent or vowel.This gave latitude to the writers to frame the language of their writings to acquire more than one meaning. For instance, kapa, tapa, papa and yapa all mean 11.

12.5 APPLICATIONS

We now take four illustrations which when decoded provide interesting mathematical insight.

(i) केवलैः सप्तकं गुणयात्

Here Saptakam means 7 and Kevalaih represents 143. We are told that in case of 7 our multiplicand should be 143.

Applying Ekanyuna corollary of Nikhilam all this means as;for 7 (as denominator) multiply 143 by 999. This is the answer for -

$$\frac{1}{7} = .\dot{1}4285\dot{7}$$

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(ii) कलौ क्षुद्रससेः

Here kalau means 13 and Kshudrasasaih represents 077. The hint here is that for $\frac{1}{13}$ multiplicand is 077, i.e. $\frac{1}{13}$ is obtained by 077 x 999.

$$\text{Thus } \frac{1}{13} = 0.\dot{0}7692\dot{3}$$

(iii) कंसे क्षामदाह खलेर्मलैः

Here Kense means 17 and ksama-daha-khalairmalaih means 05882353. Thus

$\frac{1}{17}$ is obtained by 05882353 x 99999999

i.e. $\frac{1}{17} = .0588235294117647.$

(iv) गोपी भाग्यमधव्रात - श्रृंगिशोदधिसन्धिग खल जीवित खाताव गलहालारसंधर।।

Lastly a Shloka for obtaining value of $\pi/10$.

This Shloka has two sets of literal meaning, first in praise of Lord Krishna and second for the Lord Shiva. Third meaning (after decoding) gives value of $\pi/10$ to 32 places of decimals as below

$$\frac{\pi}{10} = .31415926535897932384626433832792 \text{ ------}$$

Further, Swami ji mentions about the inbuilt key contained within this Shloka using which we can further extend the value of to any desired level of accuracy. There is a need to systematically and do research on this issue. The 32nd digit is obtained as 2 which is not the actual value, but it is rather a hint for computing the further digits. Initial study has revealed that next few digits are obtained by computing the complements of the digit already obtained. This is perfectly in tuned with the Vedic tradition of minimizing the efforts and maximization of results. We can see the similar principal at work even in the third example where all the sixteen digit of the complete recurring string of total value of 1/17 could have been easily codified but the Vedic rishi's did not believe in even an iota of extra and unnecessary work. Therefore only the necessary first 8 digits were codified. Here again the last digit is 3 (Although the correct digit is 2) which automatically gets corrected in the process of computing the

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remaining 8 digits.

Further, this raises some very serious questions in any technical and research oriented mind. It is really very difficult to imagine the extent of the accuracy of their computations for which they needed over 30 digits, which have been codified in this Shloka.

12.6 OBSERVATIONS

Dr. V.P. Dalal (of Heidelburg University Germany) after knowing above Shloka's merit commented as follows -

It shows how deeply the Ancient Indian Mathematicians penetrate, in the subtlety of their calculations, even when the Greeks had no numerals above 1000 and their multiplications were so very complex, which they performed with the help of the counting frame by adding so many times the multiplier; 7 x 5 could be done by adding 7 on the counting frame 5 times etc. etc.

12.7 FURTHER STUDY:

Further research by SSG is revealing very interesting results. For instance the sacred gaytri Mantra, when decoded yields the sacred 108 number. Further details shall be presented in future PUSHPS. All the serious students of Vedic Maths are most welcome to carry out the further research in this direction.

PROBLEMS

Q.1 Decode the following words. Use Vedic Numerical code:

(i) रमेश (ii) जयदेव (iii) हरि

(iv) माया (v) भोगेन्द्र (vi) गाँधी

(vii) भारत

Q.2 Find the meaning of the following:

i) 16 and 56 of 82 are going to 81.

ii) 32 of 4345 taught us English today.

iii) I want my 55 and 22.

Q.3 Compose suitable lines/sentences in Sanskrit/Hindi which have simple meaning and also codify the following numerical values according to the Vedic Code. Send your compositions as new links to SSG.

i) 39 - 025641 ii) 29 - 03448275862069

iii) 19 - 052631579

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CHAPTER 13

MULTIPLE CHOICE

13.1 INTRODUCTION:

We have already seen the tremendous amount of choices provided by the Vedic methods. It shall be useful to briefly review them together, although the details have been discussed in Pushp-1, 2 & 3.

13.2 NUMBERS:

We know very well that in Vedic system we can straight away look a each digit in two forms, using the complements. For example

$$6 = 10 - 4 = 1\bar{4}$$

$$2 = 10 - 8 = 1\bar{8}$$

and $38 = 40 - 2 = 4\bar{2}$ (Refer Pushp-1, C-3)

We can easily extend this and a number in general can be written in a large number of alternative ways, each of which is perfectly valid, correct and has the same value. For example

$$28394 = 3\bar{2}394 = 3\bar{2}41\bar{4}$$

$$= 3\bar{1}\bar{6}14 = 3\bar{1}\bar{6}0\bar{6}$$

$$= 1\bar{7}\bar{1}\bar{6}0\bar{6}$$

Therefore we can see that as per the requirement and convenience we can use any of these forms. Let us not conclude that these are the only forms. There seems to be absolutely no limit in number of ways we could write any number. As an example let us take a still smaller number, 38.

$38 = 4\bar{2} = 1\bar{6}\bar{2}$ are the three forms we can immediately think off, but I am sure that those of us, who have been practicing Vedic Mathematics regularly even for 15 minutes everyday(Let us be very sure that regular practice of solving a few sums in the Vedic way is necessary for activating all the neurons of human brain and training our mind in the Vedic way where we think of not 1 or 2 but the art of all possibilities) would think of another way also and then yet another possibility, followed by yet another, and another.....

$$38 = 1\bar{6}\bar{2} = 1\bar{9}\bar{6}\bar{2} = 1\bar{9}9\bar{6}\bar{2}$$

$$= 1\bar{9}9\bar{9}6\bar{2} \text{ etc.}$$

OR $= 1\bar{7}\bar{8} = 1\bar{9}\bar{7}\bar{8} = 1\bar{9}9\bar{7}\bar{8}$ etc.

Extending on these lines I am sure we shall all agree that we can have a larger number of series like this for bigger numbers and

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there are infinite number of ways in which we can write any number.

13.3 ADDITION, SUBTRACTION AND MULTIPLICATION OF NUMBERS:

A number of alternative methods for each of these elementary arithmetical operations have already been explained at different places in Pushp 1,2 and 3.

For example

```
     . .
  4 8 9 6
        .
  2 3 5 7
  .
  5 3 2 6
    . . .
    6 6 6

      4 4
      . .
  2 5 3 1
----------
1 5 8 2 0
---------
```

(i) We go on additing the digits column by column from top to bottom and using the upsutra Sudha as soon as the addition crosses 9 and put a dot above the digit just added.

ii) As an alternative we can add the complement of the bigger digits and get the same result. (Refer Ch-1). This shall be the quickest way.

iii) We can first convert the numbers in their Vinculum form and then add. (Refer Chapter-1)

iv) We may use the present way of cumulative addition of each column which is quite slow and also prone to error.

v) Fifth method would be any conveneient mixing of the above three approaches. Further, all these methods can be used for adding from bottom to top.

13.4 SQUARING:

Let us see the number of ways in which we can square number 88.

In the present system we can think of only 88 x 88.

$88^2 = 88*88$

i) Using Yavadunam upsutra (refer Ch. 6)

$88*88 = (88-12) \,/\, 12^2 = 76\ 44 = 7744$

1

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Let us remember that we do not need to multiply even 12*12 as

$$12^2 = 12 + 2/2^2 = 144$$

It shall be interesting to emphasize that to compute the square of 987 we need to multiply only 3*3 in addition to two subtractions and one addition.

ii) Using the Vinculum form alongwith Yavadunam method

$$(1\overline{12} + \overline{12})/(\overline{12})^2 = 1\overline{24}/144$$

$$= 1\overline{2}344 = 7744$$

iii) Using the Nikhilam method directly.

88 - 12 (Base = 100) (Ref. Pushp-1, Ch.-7)
88 - 12
---
76 /144 = 77 44

iv) Using Urdhva sutra from right to left.

88 (Ref. Pushp-1, Ch.-9)
88

7744 = 7744
136
---

v) Using Urdhva sutra from Left to right.

88 (Refer Pushp-2)
88

6484
126

= 7744
---

vi) Using Dwandwa Yoga upsutra. (Ref. Cha.-8)

$$88^2 = 64/8/4 = 7744$$
$$2/6$$

vii) We can also work left to right straight away working in one line.

$$88^2 = 7 /_7 /_44$$
$$/ \overline{6} / \overline{2}$$

$$= 7744$$

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We could also use the Dwandwa Yoga after converting to the Vinculum form which shall involve much lesser computation efforts.

viii) Right to Left

$$1\ \bar{1}\ \bar{2}^2 \quad = \quad 1/\bar{2}/\bar{3}/4/4 \quad = \quad 7744$$

ix) Left to Right

$$1\ \bar{1}\ \bar{2}^2 \quad = \quad 1/\bar{2}/\bar{3}/4/4 \quad = 7744$$

We can also obtain the result straight away in the positive digits by using the complement of bar digits. (Ref. Ch. 8).

x) Right to Left

$$1\ \bar{1}\ \bar{2}^2 \quad = \quad \begin{matrix} 0/_7/_7/4/4 \\ /\ 1/\ 1/\ / \end{matrix} \quad = 7744$$

xi) Left to Right

$$1\ \bar{1}\ \bar{2}^2 \quad = \quad \begin{matrix} 0\ /0\ /7\ /7\ /44 \\ 1\ \ 8\ \ 7\ \ 4 \end{matrix}$$

$$= \quad 7744$$

In addition to these eleven alternative methods we could also use the Urdhva sutra both from right to left or left to right after converting into the Vinculum form.

13.5 **DIVISION:**

Now let us look at some of the alternative working procedures for division of numbers provided by the Vedic maths sutras. For example

Divide 123192 by 108.

Since the divisor is small therefore the use of Nikhilam method of division is ruled out and we can solve this problem by using either the Paravartaya method or the Dhwajank method. But within each of these we have a large number of choices in working out the detailed solution.

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Paravartaya Method: (Refer Pushp-2 Cha.10 and Chap. 7 of Pushp -3).

i) Using the Vinculum form of divisor

```
          ___________
108 / 1 2 3 1 / 9 2
11 2̄      1̄ 2
 1̄2        1̄ 2
            (4̄ /8)*
             3̄/ 2̄
              / 0 0
          ------------------
          1 1 4 0 / 7 2
          ------------------
```

*It is much more efficient to write 48 in the alternative form of 32 (as $\overline{48} = -40 + 8 = -32 = \overline{32}$).

ii) Converting the dividend also into the Vinculum form

```
            ____________
           /
108 / 1 2 3 2 / 1̄ 2
11 2̄      1̄ 2
 1̄2        1̄ 2
             4̄ / 8*
               / 0 0
          --------------
          1 1 4 0 / 7 2
          --------------
```

*It is more useful to keep this number as obtained. However if we change, we shall end up in negative remainder and hence unnecessary task.

iii) If we retain the divisor in ordinary form

```
            ____________
           /
108 / 1 2 3 2 / 1̄ 2
 0 8̄       0 8̄
            (0 16̄)*
             1̄  6̄
               (0/48)*
                4/8
                 /0 0
          ---------------
          1 2 6̄ 0 / 7 2          =  1140/72
```


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*We can carry the excess digit to the L.H.S. column. (Ref. Ch.-7)

iv) We retain both the divisor and the dividend as they are

```
                  _____________
108 / 1 2 3 1 / 9 2
 08       0 8
          (0 16
           1  6
             (0/48)
             (4/ 8)*
              5/2
               /0 0
            -------------
            1 2 6 0 / 7 2      =  1140/72
            -------------
```

*A vigilant mind can easily see that keeping 48 as it is would lead to very big remainder and therefore it is much more efficient to wright this number in the alternative Vinculum form $48 = 5\overline{2}$.

Straight Division:

Here again we have a number of alternatives in working out the details.

v) Keeping the numbers as they are and using 10 as the divisor and 8 as the flag digit.

```
          _____________
    8 / 1 2 3 1 9 : 2 :
10        2 5 3  7
        ---------------
        1  1 4  0 :72 :
        ---------------
```

vi) If we change the dividend in Vinculum form we shall have a slight alteration in the working details.

```
          _____________
         /
    8/1 2 3 2 1 : 2 :
10
        2 5 4 7
      ---------------
       1 1 4 0  :72 :
      ---------------
```


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vii) We may also convert the divisor in Vinculum form

```
  2̄ /_____________
11  /1 2 3 2 1̄ : 2 :
      1 4 0  7
    --------------
    1 1 4 0  :72 :
    -------------
```

viii) We may retain the dividend in its original form.

```
  2̄/_____________
11 /1 2 3 1 9 : 2 :
      1 4 1̄  7
        *
    ---------------
     1 1 4 0  :72 :
    ---------------
```

*In third step it shall be convenient to use 4 as the quotient digit and $\bar{1}$ as the remainder.

ix) We may be tempted to consider only 1 as the divisor and 08 as the two flag digits.

```
  08/_____________
1 /1 2 3 1 : 9 2 :
     0 0 1 5̄
    ---------------
    1 2 5̄ 15̄
    ---------------
    1 2 6̄ 0 : 7 2 :
    ---------------
```

In Step 3 if we use only $\bar{5}$ as the third quotient digit we shall get $\overline{15}$ in the next step . Therefore it shall be convenient to take $\bar{6}$ as the third quotient digit.

We may also do it by writing the dividend into Vinculum form.

```
  12̄ /_____________
1  /1 2 3 1 : 9 2 :
     0 0 0  1̄
    ---------------
    1 1 4 0 : 7 2 :
    ---------------
```


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The remainder can be conveneintly computed in a single step. (Ref Chapter 7).

xi) We may also convert the dividend into the Vinculum form.

$$\begin{array}{r|l} \bar{1}2 & \\ 1 & 1\ 2\ 3\ 2 : \bar{1}\ 2 : \\ & 0\ 0\ 0\ 0 \\ \hline & 1\ 1\ 4\ 0 : 7\ 2 : \end{array}$$

The remainder shall be

$$\begin{array}{l} \bar{1}2 \\ 40 \quad \text{Only 1st two steps of Urdhva} \\ \hline \end{array}$$

$$\bar{8}/0$$

$$R = \bar{1}\ 2 - \bar{8}\ 0 = 72$$

Going through these eleven different ways of dividing two numbers we can see that using 1 as the divisor is slightly more challenging and hence may be avoided in the beginning.

This tremendous amount of flexibility and freedom of working provides mental relaxation which results in the frequent experience of bliss and joy. All the children in England and America who are being taught Vedic Mathematics from the very beginning are enjoying learning Maths the Vedic way. When shall we leave the unnecessary Maths anxiety from the class rooms of Mathematics in all parts of the world. You are welcome to join in this Gyan Yajna and give donation of your precious time.

PROBLEMS:

Q.1 Express the following numbers in five different forms with same number of digits.

i) 17483 ii) 26934. iii) 398272

Q.2 Express the numbers in Q.1 in the different ways with at least 8 number of digits.

Q.3 Add the following numbers in all different ways.

i) 374 + 267 + 419 + 853 + 768

ii) 2378 + 837 + 4281 + 7456

iii) 1237 + 3218 + 9457 + 87

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Q.4 Square the following numbers in Eleven different ways.

i) 87^2 ii) 9986^2 iii) 1014^2

Q.5 Divide the following in at least 10 different ways.

i) 138293 / 107

ii) 219842 / 982

iii) 3128137 / 9918

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GLIMPSES

INTRODUCTION:

The Ancient Vedic Mathematics is a vast subject and PUSHP 1,2 and 3 provide Vedic methods for Arithmetical operations. The applications in higher mathematics have already been revived in many areas.The pioneering book on Vedic Maths by Shankracharya Ji gives VM applications to algebraic operations, factorisation , simple quadratic and higher order equations, simultaneous quadratic equations, partial fractions, calculus, squaring, cubing, square root, cube root, coordinate geometry etc. etc.The recent research at Roorkee University, America and in U.K. includes applications of Vedic Mathematics to trigonometry, three dimensional coordinate geometry, solution of plain and spherical triangles, linear and non linear differential equations, matrices and determinants, log and exponentials etc.

In one of the recent five publications from U.K., Prof. K. WILLIAMS in his book, Discover Vedic Mathematics, has solved the complete mathematics papers of G.C.E. A-level and O-level examinations of U.K., thereby demonstrating that the entire mathematics upto our 10 + 2 system can be done by using Vedic methods.The three English Professors in their recent publication Vertically and Crosswise (Refer list of new books in VM News) and the author in his recent book on Higher Applications have demonstrated how the Vedic 'URDHVA SUTRA' alone can be used for solving a very large class of problems including evaluation of trigonometrical functions and their inverse, logarithm, exponentials, higher order equations, partial differential equations and integrol differential equations. Details of these are planned to be published in future PUSHPS of this PUSHPMALA. A few glimpses are presented in the following sections.

ARITHMETICAL OPERATIONS:

1. Express:- $\frac{17125}{59999}$ as a decimal upto 20 places.

By Vedic Sutra:- Ekadhikena Purvena

$$\frac{1\,7\,1\,2\,5}{5\,9\,9\,9\,9} = 0.28542142369039483991$$

2. Solve the fraction $\frac{2\,4\,3}{1\,6\,0\,1}$

$$\frac{243}{1601} = .151780137414116179$$

3. Find square root .0009240160 upto 5 places of decimals.

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```
6 : 00 : 09 : 2 4 0 1 6 0
  :    :    : 0 2 6 6
-------------------------
  :  0 :  3 : 0 3 9 7 ....
```

4. Find the cube root of 33, 076, 161 in three steps we get

```
331 0761 161
            1
--------------
33   076   16
           6
--------------
33 0761
```

Cube root = 321

ALGEBRAIC APPLICATIONS

5. Find Quotient and Remainder in following division, by Vedic Sutra- Paravataya-Yojayet.

$$\frac{x^3 - x^2 - 7x + 4}{x - 3}$$

Quotient = $x^2 + 2x - 1$ Remainder =1

6. Solve Equation : By Vedic Sutra - Shunyam Samya Samuchchya.

$$\frac{1}{x+3} + \frac{1}{x-8} = \frac{1}{x+4} + \frac{1}{x-9}$$ Answer x = 5/2

URDHVA SUTRA APPLICATIONS:

Recently the Urdhva Sutra has been applied to solve a large class of problems. (Refer List of Books).

7. **Determinants**

Evaluate -

$$\begin{vmatrix} 2 & 6 & 7 \\ 5 & 4 & 8 \\ 3 & 1 & 9 \end{vmatrix}$$

-7 -21 28 = -119

Here Urdhva Sutra is used.

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8. **Solution of Simultaneous Equations:**

-8	4x-2y	+3z	=8	-5
14	2x-3y	+ z	=1	-17
13	3x+2y	- z	=3	4

. . D =33; x.D =33; -y.D = -33; z.D =66

where x =y = 1; z =2. Sutra (No.3) is used.

9. **Curve Fitting:**

Find a quadratic passing through 3-points (8,7), (6,5) & (9,4) Changing origin to (6,5), we get (2,2), (0,0) & (3, -1)

If equation is $Y = AX^2 + BX$

(or) $6y = -8x^2 + 118x - 390$

10. Evaluate using Urdhva Sutra

B = exp(23.51)

$B = \exp(23.51) = 1.62279 \times 10^{10}$

11. Find the roots of the biquadratic equation

$x^4 - 20x^3 + 137x^2 - 382x + 360 = 0$

using lopanasthapana sutra in four steps we get

x =5, 4, 2 or 9.

These are just a few stray examples & illustrations only.

TRIBHUJANK:

Tribhujank is a very interesting, useful and thought provoking ancient knowledge. A set of three numbers x, y and z which satisfy the condition $x^2 + y^2 = z^2$ is called Tribhujank. Two Tribhujanks can be added/subtracted by using the Urdhva sutra (No.3). The working of problems Trigonometry both Plane and Spherical as well as Co-ordinate Geometry (2D and 3D) is drastically simplified. The number of formulae used at present become largely redundent. Some applications of Tribhujanks in these areas are available else where. Here we just consider one example from Loni's Trigonometry.

12. Prove that $\sin 105^\circ + \cos 105^\circ = \cos 45^\circ$.

We know that Tribhujank for:

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45^{o}	1	1	$\sqrt{2}$
60	1	$\sqrt{3}$	2
Adding $60^{o}+45^{o}$ $=105^{o}$	$1-\sqrt{3}$	$1+\sqrt{3}$	$2\sqrt{2}$

Therefore substituing we get

$$\frac{1+\sqrt{3}}{2\sqrt{2}} + \frac{1-\sqrt{3}}{2\sqrt{2}} = \frac{1}{\sqrt{2}} = \cos 45^{o} \quad \text{Proved.}$$

In just one line.

The present method would require the following steps:

$$\begin{aligned}
\sin 105^{o} + \cos 105^{o} &= \sin(180^{o} - 105^{o}) - \cos(180^{o} - 105^{o}) \\
&= \sin 75^{o} - \cos 75^{o} \\
&= \sin 75^{o} - \sin(90^{o} - 75^{o}) \\
&= \sin 75^{o} - \sin 15^{o} \\
&= 2\cos 45^{o} \sin 30^{o} \\
&= 2\cos 45^{o} * \frac{1}{2} \\
&= \cos 45^{o}
\end{aligned}$$

CALCULUS:

13. Find the third derivative of the product $x^{6} . e^{3x}$.

Using Urdhva sutra we obtain the result in two steps.

$$D3 = 27x^{6} e^{3x} + 162 x^{5} e^{3x} + 270 x^{4} e^{3x} + 120 x^{3} e^{3x}$$

Further details are available in Higher Applications of VM.

It is possible to solve a class of non linear differential, Integro differential and even Partial differential equations which otherwise required the use of iterative procedures. This is due to the unique facility of carrying out every operations from left to right.

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MULTIPLICATION - LEFT TO RIGHT:

Even the left to right multiplication can be carried out directly in one line by using the unique Vedic Method of shifting to the left. Further it is also possible to obtain the results directly in positive digits. Although the numbers being operated may be in Vinculum form. Details of these unique features of VM are being presented in the revised and enlarged 2nd edition of Pushp-2.

14. Multiply 27819 * 21898

Using Vinculumn form

$3\ \bar{2}\ \bar{2}\ 2\ \bar{1}$

$2\ 2\ \bar{1}\ 0\ \bar{2}$

$6\ 0\ 9\ 1\ 8\ 0\ 4\ 6\ 2$

This result can be obtained by working from left to right or right to left as convenient.Frequently we may need only the first four digits, then VM provides the unique approach and we would require less than half of mental working.

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ANSWERS

CHAPTER 1

Q.1 i) 10822 ii) 14001896 iii) 20018

Q.2 i) 268 ii) 358 iii) 1343

CHAPTER 2

Q.1 i) 1881 ii) $\overline{2}$ 207 iii) 21189

iv) 245416 v) -5246

Q.2 Same as Q. 1.

CHAPTER 3

Q.1 i) 18791 ii) 96324 iii) 9824715

iv) 98799122 v) 1568 vi) 999979964

vii) 11187568 viii) 10880978 ix) 988911

x) 1242669

CHAPTER 4

Q.1 i) 5472 ii) 57525 iii) 165509

iv) 633390 v) 1084541 vi) 10634832

vii) 301875 viii) 15682104 ix) 461728

x) 5959104 xi) 2064510

CHAPTER 5

Q.1 149742 same for both parts.

Q.2 170011520

Q.3 i) 15747732 ii) 13300 iii). 168789978

iv) 15426048 v) 138966

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CHAPTER 6

Q.1 i) 51529 ii) 180625 iii) 670761
iv) 53816896 v) 24870169 vi) 45996880410
viii) 8540717056

Q.2 i) 99840064 ii) 100120036 iii) 10003800361
iv) 123254404 v) 99780121 vi) 9999400009
vii) 123543225

Q.3 i) 2116 ii) 63504 iii) 168921
iv) 474721 v) 183184 vi) 312481
vii) 17935225

Q.4 Same as above.

CHAPTER 7

Q.1 i) Q = 110, R = 131 ii) Q = 338; R = 156
iii) Q = 621; R = 165

Q.2 i) Q = 204; R = 65 ii) Q = 243; R = 231
iii) Q = 6520; R = 31

Q.3 i) Q = 23.616 ii) Q = 141.9885
iii) Q = 180.856

Q.4 i) Q = 737; R = 101 ii) Q = 613; R = 563
iii) Q = 998; R = 85

Q.5 i) 0.00001043 ii) 0.31982 iii) 127.915

CHAPTER 8

Q.1 i) 1849 ii) 5696 iii) 13667809
iv) 321489 v) 5499025 vi) 21547304
vii) 72134142084 viii) 207762690513664

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ix) 187190214336

Q.2 i) 974169 ii) 7230721 iii) 4747485604

iv) 3202729241190225 v) 10867479255396

Q.3 i) 103175.8641 ii) 16225.6644 iii) 589.469841

Q.4 i) .053361 ii) .00141376

iii) .0000060025

CHAPTER 9

Q. 1 i) 6859 ii) 17576 iii) 226981

iv) 2248091 v) 79507 vi) 2097152

vii) 704969 viii) 1012048064 ix) 941192

x) 996703528779 xi) 100045067503375

Q.2 Same as Q.1.

CHAPTER 10

Q.1 i) 39402.321 ii) 867.8294 iii) 2722.27

iv) 89494.358 v) 1372.0076 vi) 27183

vii) 5074 viii) 6732 ix) 31377.05

x) 9640.81 xi) 1166.61

CHAPTER 11

Q.1 Area is 27 sq.m 97 sq.cm

Total cost is Rs. 88.07.

Q.2 i) Area = 20sq foot 44 sq inch

Total cost = Rs. 314.736

ii) Area = 52 sq.foot 102 sq inch

Total cost = Rs. 816.98

iii) Area = 41 sq foot 25 sq inch

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Total cost = Rs. 638.19

iv) Area = 21 sq yd 2 sq foot 27 sq inch

Total cost = Rs. 2963.41

Q.3 Rs. 2284.86

Q.4 25 km 416 m.

CHAPTER 12

Q.1 i) 255 ii) 8184 iii) 82

iv) 51 v) 138 vi) 37

vii) 426

Answer for remaining questions shall sent to SSG after solving. Gift may be given for every good answer.

CHAPTER 13

Q.1 i) $1\ 7\ 4\ 9\ \overline{7}$, $1\ 7\ 5\ \overline{1}\ \overline{7}$, $1\ 8\ \overline{5}\ \overline{1}\ \overline{7}$, $2\ \overline{2}\ \overline{5}\ \overline{1}\ \overline{7}$

$2\ \overline{2}\ \overline{5}\ \overline{2}\ 3$ etc.

ii) $2\ 6\ 9\ 4\ \overline{6}$, $2\ 7\ 0\ \overline{6}\ \overline{6}$, $3\ \overline{3}\ 0\ \overline{6}\ \overline{6}$, $3\ \overline{3}\ 0\ \overline{7}\ 4$

$3\ \overline{4}\ 9\ 4\ \overline{6}$ etc.

iii) $3\ 9\ 8\ 2\ 8\ \overline{8}$, $3\ 9\ 8\ 3\ \overline{2}\ \overline{8}$, $3\ 9\ 9\ \overline{7}\ \overline{2}\ \overline{8}$, $4\ 0\ \overline{1}\ \overline{7}\ \overline{2}\ \overline{8}$

$4\ 0\ \overline{1}\ \overline{7}\ \overline{3}\ 2$ etc.

Q.2 i) $1\ \overline{9}\ \overline{9}\ \overline{8}\ \overline{2}\ \overline{5}\ \overline{2}\ \overline{3}$ etc.

ii) $1\ \overline{9}\ \overline{9}\ \overline{7}\ \overline{4}\ 9\ 4\ \overline{6}$ etc.

iii) $1\ \overline{9}\ \overline{9}\ \overline{6}\ 0\ \overline{1}\ \overline{7}\ \overline{3}\ \overline{2}$ etc.

Q.3 i) 2681 ii) 14952 iii) 13999

Q.4 i) 7569 ii) 99720196 iii) 1028196

Q.5 i) Q = 1292, R = 49 ii) 223 ,R = 856

iii) Q = 315, R = 3967

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FUTURE OF VEDIC MATHEMATICS

Future Research : There are almost infinite number of applications of Vedic Mathematics in different branches of science and technology. Prof. Andrew Nicholas of Polytechnic of North London foresees very bright future for Vedic Mathematics. In addition to the topics already listed under the VM News, Revered Bharti Krishen Tirthji Maharaj has mentioned about the Vedic methods for solving problems of interest, Statics, Dynamics, Hydrostatics, Pneumatics, Applied mechanics, Solid Geometry, Plain and Spherical Trignometry and Astronomy etc.etc. Dr. Verma of Delhi has recently informed that Revered Swamiji had delivered a series of 12 lectures on Vedic Mathematics for Fluid dynamics aspects of Aeronautics.

It seems that what is available today in Vedic Mathematics is only a small visible tip above the water surface, of the huge ice berg floating in sea.

These require dedicated research work to rediscover the hidden treasures.

The feedback from those who have learnt Vedic Mathematics in a nutshell brings out that :-

1. VM converts the tedious mathematics into an interesting and playful subject, which children learn with smiles. Therefore Vedic Maths has the inherent qualities to eliminate the problem of 'MATHS ANXIETY' being faced in Maths education, all over the world.

Exactly same has been the response of hundreds of the American children who have started learning Vedic Mathematics regularly in their classes after the recent visits of the author to America. All of them are enjoying the bliss of learning Vedic Maths and their maths classes have no longer the routine tense and dry atmosphere of a maths class. Their education directer in the first report informed that the children are enjoying and bubbling up with bliss and joy in Vedic Maths class.

2. Participating teachers and children often ask why VM is not being taught in every school and college, all over the world.

3. Vedic Maths system has the capacity for the **HOLISTIC DEVELOPMENT OF THE HUMAN BRAIN.**

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Recently, after teaching one sutra of Vedic Maths when the teacher wrote down a sum on the blackboard in an American school at Fairfield (IOWA), one student immediately shouted out the answer. When the teacher asked him,"how did you get at that so soon". The student innocently replied, "Sir it just flashed in my mind". That is the working of the intuition faculty.

This task of developing and spreading Vedic Mathematics in all parts of the world requires the dedicated efforts of the lovers of the humanity, who are concerned about the difficulty, the students are facing in learning Maths and who have 'Shradha' for Ancient Vedic Cultural heritage,which is not only the oldest but also the noblest.

ANCIENT VEDIC MATHEMATICS

PUSHP MALA:- With a view to provide the comprehensive details of this wonderful Ancient Bhartiya Cutural Heritage SSS is planning to publish the PUSHP MALA of Vedic Maths. PUSHP -1 and 2 have already been published in English and Hindi.

The SSS most prayerfully and humbly is publishing the PUSHP -3 in English. Further, higher applications of VM have also been published.

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INVITATION

You are most cordially invited to participate in this 'Gyan Yagya'(homa of knowledge) by giving Ahuti (donation) of your precious time. You can do as many of the following as convenient.

- **Spreading the news of Vedic Mathematics.**

* Chatting about VM when you are amongst unknown people.
* Giving small illustrations of the superfast cases to interested students, teachers and persons.
* Informing all that Vedic Maths is a complete scientific system. By regularly practising VM the total holistic development of the Human brain automatically takes place. Further the development of the most powerful facet of human personality namely INTUITION steadily takes place.
* Giving demonstration lectures in your near by institutions.As a result of which you can yourself enjoy the upsurge of bliss in your heart by repeatedly observing the blissful and smiling faces of the learners.
* Organising the short VM courses at your place.

- **Spreading the Vedic Mathematics Literature.**

* Gifting VM books to your kith and kin.
* Gifting VM literature to the libraries in your area.
* Gifting VM books as prizes at the functions, you organise

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- **Developing Vedic Mathematics.**

* Collecting the ancient superfast computational procedures and literature and sending to/or/informing SSG.
* Applying Vedic Sutras to solve the problems of different sciences and sending to SSG for compilation and publication.
* Meditating on Vedic Sutras and further developing them and their applications in spiritual, metaphysical and psychological levels.

SSG is meant to assist you in this 'Yagya'. This activity according to Vedanta shall decidedly help us in cleaning our mind and then the SELF within shall automatically stand revealed to the self.

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HOLISTIC DEVELOPMENT OF HUMAN BRAIN

According to the brain specialists the left half of the brain, systematically collects the information, does sequential analysis and arives at the logical conclusions and results. Only left half of the brain is activated and developed by the present Maths education, being given in the schools and colleges.The right half of the human brain works with pattern recognition and has the capability of intuition. This most powerful facet of human personality - the intuitive faculty, unfortunately, remains undeveloped in most of the students. In Vedic Mathematics system, the very first step is to recognize the pattern of the problem and pick up the most efficient Vedic algorithm. Further, at each subsequent step, we recognize the pattern and complete the task by using the appropriate superfast, mental working procedure.As in Vedic Maths we have multiple choices available at each stage of working. This practice, systematically develops the right half of the brain as well. As such by the regular practice of the most natural mental procedures of Vedic Maths system, the holistic development of the human brain automatically takes place. In a nutshell we can say that the Vedic Maths provides the Cosmic software for the Cosmic computer (the human mind), so that the later is not wasted.

MATHEMATICS WITH SMILES

Vedic Mathematics has originated from the Ved.Ved is all divine knowledge. Any knowledge derived from the Ved is bound to have the touch of the divine bliss. Therefore, the very natural, easy and superfast algorithms of Vedic Mathematics bring an upsurge of joy and bliss. Due to its very special and universal features. Vedic Maths system converts the dry and tedious Maths into a playful and joyful subject, which children learn with smiles.

COST

Paper Rs. 20/-

Bound Rs. 30/-

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